16. The vertical structure of a column of dry air is given by the following points:

1000 mb (ground)	−15.0 °C
900 mb	−10.0 °C
800 mb	−13.0 °C.

After a certain time, this column has changed to:

1000 mb (ground)	+5.0 °C
900 mb	−2.0 °C
800 mb	−7.0 °C.

The atmosphere above 800 mb is assumed to preserve its temperature stratification during the process.

(a) Compute with a diagram the initial and final thicknesses, in gpm, by any graphical method.

(b) How much have the isobars of $\leqslant 800$ mb been lifted?

(c) Compute the changes in internal energy and in potential energy (per unit cross section), and the total amount of energy that must have been absorbed by the column, in order to undergo this change.

17. In Section 17 some numerical results were given for the overturning of two 100 mb-thick layers, starting at 1000 mb, with potential temperatures 300 K and 290 K, according to Margules. Check the values obtained for U, ΔU, ΔK and W.

18. At the end of Section 19 a numerical example was given for a simple situation. Perform, for that example, the calculation of the total potential energy of enthalpy H and available potential energy A.

APPENDIX I

TABLE OF PHYSICAL CONSTANTS

Molecular weights.

$M_d = 28.964\ (\text{g mol}^{-1});\ M_v = 18.015\ (\text{g mol}^{-1})$

Gas constants

$R^* = 8.314\ \ \text{J mol}^{-1}\text{K}^{-1};\qquad \varepsilon = R_d/R_v = 0.622$
$R_d = 287.05\ \text{J kg}^{-1}\text{K}^{-1};\qquad R_v = 461.51\ \text{J kg}^{-1}\text{K}^{-1}$

Specific heat capacities

Dry air (see also Table II-1)	$c_{p_d} = 1005\ \text{J kg}^{-1}\text{K}^{-1} = 0.240\ \text{cal g}^{-1}\text{K}^{-1}$ $c_{v_d} = \ \ 718\ \text{J kg}^{-1}\text{K}^{-1} = 0.171\ \text{cal g}^{-1}\text{K}^{-1}$ $\varkappa_d = 0.286;\qquad \eta_d = 1.40$
Water vapor	$c_{p_v} = 1850\ \text{J kg}^{-1}\text{K}^{-1} = 0.443\ \text{cal g}^{-1}\text{K}^{-1}$ $c_{v_v} = 1390\ \text{J kg}^{-1}\text{K}^{-1} = 0.332\ \text{cal g}^{-1}\text{K}^{-1}$
Water, at 0 °C (see other values in table below)	$c_w = 4218\ \text{J kg}^{-1}\text{K}^{-1} = 1.008\ \text{cal g}^{-1}\text{K}^{-1}$
Ice, at 0 °C	$c_i = 2106\ \text{J kg}^{-1}\text{K}^{-1} = 0.503\ \text{cal g}^{-1}\text{K}^{-1}$

Critical and triple point constants of water: see Chapter IV, Section 6.

Latent heats of phase transformations of water, at 0 °C (see other values in table below)

$l_f = 0.334 \times 10^6\ \text{J kg}^{-1} = \ \ 79.7\ \text{cal g}^{-1}$
$l_v = 2.501 \times 10^6\ \text{J kg}^{-1} = 597.3\ \text{cal g}^{-1}$
$l_s = 2.834 \times 10^6\ \text{J kg}^{-1} = 677.0\ \text{cal g}^{-1}$

Thermodynamic properties of condensed water (see also Chapter IV, Table IV-5)

t °C	c_i J kg^{-1} K^{-1}	e_i mb	l_s 10^6 J kg^{-1}	l_f 10^6 J kg^{-1}	l_v 10^6 J kg^{-1}	e_w mb	c_w J kg^{-1} K^{-1}
−100	1382	1.402×10^{-5}	2.824				
−90	1449	9.665×10^{-5}	2.828				
−80	1520	5.468×10^{-4}	2.832				
−70	1591	2.614×10^{-3}	2.834				
−60	1662	1.080×10^{-2}	2.837				
−50	1738	3.933×10^{-2}	2.8383	0.2035	2.6348	0.06354	5400
−40	1813	0.1283	2.8387	0.2357	2.6030	0.1891	4770
−30	1884	0.3797	2.8387	0.2638	2.5749	0.5087	4520
−20	1959	1.032	2.8383	0.2889	2.5494	1.254	4350
−10	2031	2.597	2.8366	0.3119	2.5247	2.862	4270
0	2106	6.106	2.8345	0.3337	2.5008	6.107	4218
5					2.4891	8.718	4202
10					2.4774	12.27	4192
15					2.4656	17.04	4186
20					2.4535	23.37	4182
25					2.4418	31.67	4180
30					2.4300	42.43	4179
35					2.4183	56.23	4178
40					2.4062	73.77	4178
45					2.3945	95.85	4179
50					2.3823	123.39	4181

Saturation vapor pressures over pure liquid water (e_w) and over pure ice (e_i) as functions of temperature
Detailed table.

T(°C)	e_w (mb)	e_i (mb)	T	e_w	e_i	T	e_w	T	e_w
−50	0.0635	0.0393	−24	0.8826	0.6983	1	6.565	26	33.606
−49	0.0712	0.0445	−23	0.9647	0.7708	2	7.054	27	35.646
−48	0.0797	0.0502	−22	1.0536	0.8501	3	7.574	28	37.793
−47	0.0892	0.0567	−21	1.1498	0.9366	4	8.128	29	40.052
−46	0.0996	0.0639	−20	1.2538	1.032	5	8.718	30	42.427
−45	0.1111	0.0720	−19	1.3661	1.135	6	9.345	31	44.924
−44	0.1230	0.0810	−18	1.4874	1.248	7	10.012	32	47.548
−43	0.1379	0.0910	−17	1.6183	1.371	8	10.720	33	50.303
−42	0.1533	0.1021	−16	1.7594	1.505	9	11.473	34	53.197
−41	0.1704	0.1145	−15	1.9114	1.651	10	12.271	35	56.233
−40	0.1891	0.1283	−14	2.0751	1.810	11	13.118	36	59.418
−39	0.2097	0.1436	−13	2.2512	1.983	12	14.016	37	62.759
−38	0.2322	0.1606	−12	2.4405	2.171	13	14.967	38	66.260
−37	0.2570	0.1794	−11	2.6438	2.375	14	15.975	39	69.930
−36	0.2841	0.2002	−10	2.8622	2.597	15	17.042	40	73.773
−35	0.3138	0.2232	−9	3.0965	2.837	16	18.171	41	77.798
−34	0.3463	0.2487	−8	3.3478	3.097	17	19.365	42	82.011
−33	0.3817	0.2768	−7	3.6171	3.379	18	20.628	43	86.419
−32	0.4204	0.3078	−6	3.9055	3.684	19	21.962	44	91.029
−31	0.4627	0.3420	−5	4.2142	4.014	20	23.371	45	95.850
−30	0.5087	0.3797	−4	4.5444	4.371	21	24.858	46	100.89
−29	0.5588	0.4212	−3	4.8974	4.756	22	26.428	47	106.15
−28	0.6133	0.4668	−2	5.2745	5.173	23	28.083	48	111.65
−27	0.6726	0.5169	−1	5.6772	5.622	24	29.829	49	117.40
−26	0.7369	0.5719	0	6.1070	6.106	25	31.668	50	123.39
−25	0.8068	0.6322	–	–	–	–	–	–	–

Saturation mixing ratio with respect to water.

Saturation (with respect to water) adiabatic lapse rate.

t °C	r_w (g kg^{-1})				γ_w (K gpkm^{-1})	
	1000 mb	800 mb	600 mb	400 mb	1000 mb	500 mb
−40	0.118	0.148	0.197	0.295	9.78	9.54
−30	0.318	0.397	0.529	0.794	9.41	8.90
−20	0.785	0.980	1.31	1.96	8.79	7.85
−10	1.79	2.24	2.99	4.49	7.80	6.50
0	3.84	4.80	6.41	9.66	6.59	5.17
10	7.76	9.72	13.02	19.73	5.39	4.11
20	14.95	18.79	25.29	38.69	4.40	3.35
30	27.69	34.98	47.50	74.02	3.66	2.87
40	49.81	63.49	87.56	141.2	3.15	2.55

Surface tension of water against air

$t = -10\,°\mathrm{C}, \quad \sigma = 0.07729\ \mathrm{Nm}^{-1}$
$t = 0\,°\mathrm{C}, \quad \sigma = 0.07570\ \mathrm{Nm}^{-1}$
$t = 20\,°\mathrm{C}, \quad \sigma = 0.07275\ \mathrm{Nm}^{-1}$

Lapse rates $\gamma_d = 9.76\ \mathrm{K\ gpkm}^{-1}$; γ_w: see table above.

Gravity

Normal acceleration of gravity $g_0 = 9.80665\ \mathrm{m\ s}^{-2}$
(other values = see Chapter VIII, Section 1)
Gravitational constant $G = 6.67 \times 10^{-11}\ \mathrm{Nm}^2\ \mathrm{kg}^{-2}$
Mass of earth $M_e = 5.98 \times 10^{24}$ kg
Angular velocity of earth $\omega = 7.292 \times 10^{-5}\ \mathrm{s}^{-1}$

Standard atmosphere: see Chapter VIII, Table VIII-1

Conversion factors

$1\ \mathrm{atm} = 1.01325 \times 10^5\ \mathrm{Pa}$
$1\ \mathrm{mb} = 10^2\ \mathrm{Pa} = 10^3\ \mathrm{dyn\ cm}^{-2}$
$1\ \mathrm{IT\ cal} = 4.1868\ \mathrm{J} = 4.1868 \times 10^7\ \mathrm{erg}$
$1\ \mathrm{gpm} = 9.80665\ \mathrm{J\ kg}^{-1} = 0.980665\ \mathrm{dyn\text{-}m}$

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Very few books have been devoted specifically to the Thermodynamics of the Atmosphere, and some are old and unavailable. The subject is, however, treated, although with little development in general, in many textbooks devoted to General and Dynamic Meteorology, or in comprehensive compendia or handbooks. We have collected a bibliography on atmospheric thermodynamics and related subjects, classified according to the type of publication, hoping that this may give some orientation for further reading. Alphabetical order within each section has been followed. No claim is made for exhaustiveness.

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ANSWERS TO PROBLEMS

Chapter I

1. $t' = 12°$. **2.** (a) $1\ \mathrm{m\,s^{-2}} = 10^2\ \mathrm{cm\,s^{-2}}$; $1\ \mathrm{kg\,m^{-3}} = 10^{-3}\ \mathrm{g\,cm^{-3}}$; $1\ \mathrm{N} = 10^5\ \mathrm{dyn}$; $1\ \mathrm{Pa} = 10\ \mu\mathrm{bar}$; $1\ \mathrm{J} = 10^7\ \mathrm{erg}$; $1\ \mathrm{J\,kg^{-1}} = 10^4\ \mathrm{erg\,g^{-1}}$. **3.** 17 atm. 5. $\overline{M} = 28.90\ \mathrm{g\,mol^{-1}}$ $\bar{R} = 287.68\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

Chapter II

1. (a) $A = nR^*T\,ln(p_f/p_i)$ (n = number of moles); (b) $A = p_f(V_i - V_f) = V_i(p_f - p_i)$; (c) $A = nC_pT[(p_f/p_i)^{\varkappa} - 1]$; (d) $A = nR^*T(p_f/p_i - 1) = V_i(p_f - p_i) = p_f(V_i - V_f)$. $Q = -A$ in all processes. **2.** (a) $v_i = 0.813\ \mathrm{m^3\,kg^{-1}}$. (b) $T_f = 256\ \mathrm{K}$; $v_f = 1.049\ \mathrm{m^3\,kg^{-1}}$. (c) $\Delta u =$ $= -2.0 \times 10^4\ \mathrm{J\,kg^{-1}}$; $\Delta h = -2.8 \times 10^4\ \mathrm{J\,kg^{-1}}$. (d) $-A = 2.4 \times 10^{13}$ J. (e) The same as in (c) and 0, respectively. (*f*) $v_i = 0.59\ \mathrm{m^3\,kg^{-1}}$; $T_f = 245\ \mathrm{K}$; $v_f = 0.73\ \mathrm{m^3\,kg^{-1}}$; $\Delta u =$ $= -1.2 \times 10^4\ \mathrm{J\,kg^{-1}}$; $\Delta h = -2.0 \times 10^4\ \mathrm{J\,kg^{-1}}$. **3.** (a) $T_{B_f} = 685\ \mathrm{K}$. (b) $A_B = 230$ J. (c) $T_{A_f} = 4777\ \mathrm{K}$; (d) $Q_A = 653$ cal. **4.** (a) $n = 1.262$. (b) $T_f = 205$ K. (c) $\Delta U = -1421$ J. (d) $A = -2168$ J. (c) $Q = 178$ cal. **5.** (a) $\Delta u = 574\ \mathrm{cal\,g^{-1}}$. (b) $\Delta h = 606\ \mathrm{cal\,g^{-1}}$. (c) $c_{p_v} = 0.44\ \mathrm{cal\,g^{-1}\,K^{-1}}$.

Chapter III

1. $\Delta S = 5.76\ \mathrm{J\,K^{-1}}$; $\Delta G = -1729$ J. **2.** $\int_i^f \delta Q/T = \Delta S = -nR^*\ \ln\ (V_i/V_f)$ for processes (a), (c), (d). (b) $\int_i^f \delta Q/T = -p_f(V_i - V_f)/T < \Delta S$. **3.** $\Delta U = \Delta H = -747$ cal; $\Delta S = -2.72\ \mathrm{cal\,K^{-1}}$; $\Delta G = -28$ cal. At the triple point. **4.** (a) $\mathrm{D}S \leqslant 0$. (b) $\mathrm{D}G \geqslant V\mathrm{D}p$; $\mathrm{D}F \geqslant -p\mathrm{D}V$. (c) $\mathrm{D}G \geqslant 0$. **5.** $\Delta h = 4.1 \times 10^{-4}\ \mathrm{cal\ g^{-1}}$; $\Delta u =$ $= 6.9 \times 10^{-6}\ \mathrm{cal\,g^{-1}}$. **6.** (a) $\Delta h = -1.23 \times 10^{-2}\ \mathrm{cal\,g^{-1}}$; $\Delta s = -7.3 \times 10^{-7}\ \mathrm{cal\,g^{-1}\,K^{-1}}$. (b) $\Delta h = -10.1\ \mathrm{cal\,g^{-1}}$; $\Delta s = -3.8 \times 10^{-2}\ \mathrm{cal\,g^{-1}\,K^{-1}}$. **7.** +11‰. **8.** It decreases. **9.** $p = 300.2$ mb; $\theta = 331.5$ K. **10.** $\mathrm{d}G < -S\,\mathrm{d}T + V\,\mathrm{d}p = 0$. **11.** (iii), which corresponds to reversible freezing at 0 °C.

Chapter IV

1. 0.695 atm. **2.** $592\ \mathrm{cal\,g^{-1}}$. **3.** $-7.5 \times 10^{-3}\ \mathrm{K\,atm^{-1}}$. **4.** (a) $\Delta S = 39.4\ \mathrm{cal\,mol^{-1}\,K^{-1}}$. (b) $-4.1\ \mathrm{cal\,mol^{-1}\,K^{-1}}$. **5.** $80\ \mathrm{cal\,g^{-1}}$. **6.** $e = 21.2$ mb; $r = 0.0135$; $q =$ $= 0.0133$; $c_p = 0.243\ \mathrm{cal\,K^{-1}\,g^{-1}}$; $T_v = 32.5\ °\mathrm{C}$; $\varkappa = 0.285$; $\theta = 30\ °\mathrm{C}$, $\theta_v = 32.5\ °\mathrm{C}$, both constant for adiabatic expansion. **7.** $T_v = 2.6\ °\mathrm{C}$; $R = 287.5\ \mathrm{J\,kg^{-1}\,K^{-1}}$; $c_p =$ $= 1007\ \mathrm{J\,kg^{-1}\,K^{-1}}$; $\varkappa = 0.286$. **8.** (a) $e = 8.6$ mb. (b) $r = 5.4\ \mathrm{g\,kg^{-1}}$. (c) $e = 6.9$ mb; r:

the same as before. **9.** 7.7 g kg^{-1}. **10.** 1.10. **11.** (1) $\Delta h = -11.3$ cal g^{-1}; $\Delta s = -0.040$ cal g^{-1} K^{-1}. (2) (a) $+0.7\%$ and $+0.7\%$, respectively; (b) -2.6% and -2.6%; (c) -1.4% and -1.4%. **12.** We assume thermal and mechanical equilibrium. Therefore T and p are uniform for all phases.

In order to determine the composition of each phase, we must know the molar ratios of all components minus one (the last one being determined by difference). This gives a total of $\varphi(c-1)$ variables of composition, where φ is the number of phases and c the number of components.

The total number of variables (T, p and composition variables) is then

$$2 + \varphi(c-1).$$

The values of these variables are restricted by the conditions of chemical equilibrium, i.e., that the chemical potential of each component must be equal in all phases:

$$\mu_{i1} = \mu_{i2} = \dots = \mu_{i\varphi} \quad (i = 1, 2, \dots c).$$

This gives $(\varphi-1)$ equations for each of the c values of i; therefore, a total of $c(\varphi-1)$ conditions.

The number of independent variables or *variance* v of the system will be the difference between the total number of variables and the restrictive conditions:

$$v = 2 + \varphi(c-1) - c(\varphi-1) = 2 + c - \varphi.$$

Chapter V

1. (a) 0.12 μm; (b) 1.20 μm; (c) 2.40 μm. **2.** (a) 1.9 μm; (b) 3.3 μm. **3.** (a) 31.7 mb; (b) 0.19%.

Chapter VI

1. After drawing the basic grid, notice that the intersections $\theta = T$ define $p = 1000$ mb. Apply the congruency property to draw the other isobars. Saturation mixing ratio lines are defined by $p = \varepsilon e_w(T)/r_w + e_w(T)$. **3.** $J = -c_p/p^{\varkappa} \neq \text{const.}$ **4.** $r = 11.3$ g kg^{-1}; $r_w = 16.2$ g kg^{-1}; $\theta = 28.5\,°$C; $T_d = 14.2\,°$C. **5.** $\Delta u = -7.17 \times 10^3$ J kg^{-1}; $\Delta h = -1.005 \times 10^4$ J kg^{-1}; $\Delta s = 10.2$ J kg^{-1}K^{-1}; $a = -1.00 \times 10^4$ J kg^{-1}; $q = 2.85 \times 10^3$ J kg^{-1}.

Chapter VII

1. $e_i = 15.8$ mb; $e_f = 11.9$ mb; $U_{w_i} = 68\%$; $U_{w_f} = 99\%$; $T_{d_f} = 9.8\,°$C. **2.** $T_f = 8.1\,°$C; $\Delta e_w = -1.5$ mb; $\Delta c = 1.2$ g m^{-3}. **3.** At every instant, $m_t = m_v$.

$$(m_d c_{pd} + m_v c_w)\,\mathrm{d}T + \mathrm{d}[l_v(T) m_v] = 0.$$

When integrating, the variation of m_v has to be taken into account:

$$\frac{\mathrm{d}T}{l_\mathrm{v}(T)} + \frac{\mathrm{d}m_\mathrm{v}}{m_\mathrm{d}c_{\mathrm{p_d}} + m_\mathrm{v}c_{\mathrm{p_v}}} = 0.$$

Introducing Kirchhoff's law: $\mathrm{d}l_\mathrm{v}(T) = (c_{\mathrm{p_v}} - c_\mathrm{w})\,\mathrm{d}T$, we have

$$\frac{1}{c_{\mathrm{p_v}} - c_\mathrm{w}}\,\mathrm{d}\ln l_\mathrm{v}(T) + \frac{1}{c_{\mathrm{p_v}}}\,\mathrm{d}\ln(m_\mathrm{d}c_{\mathrm{p_d}} + m_\mathrm{v}c_{\mathrm{p_v}}) = 0.$$

Integration gives the desired expression. **4.** $e' - e = -(c_\mathrm{p}/\varepsilon l_\mathrm{v})p(T' - T)$; $T_\mathrm{iw} = 3.3\,^\circ\mathrm{C}$; $T_\mathrm{ie} = 15.4\,^\circ\mathrm{C}$. **5.** $T = 10.6\,^\circ\mathrm{C}$; $r = 8.1\ \mathrm{g\,kg^{-1}}$; it is saturated and the liquid water content is $0.9\ \mathrm{g\,m^{-3}}$. **6.** $1.3\ \mathrm{g\,m^{-3}}$. **7.** Yes (cf. Equation (78)). **8.** Use formula (85) starting from a chosen point ($p = 1000$ mb and $T = 250, 270, 290$ for the three curves) to fix the constant. **9.** 286 mb. **10.** $-79\,^\circ\mathrm{C}$. **11.** The process occurs descending along the vapor pressure curve, say from an initial point A to a final point B (both on the curve). The second path consists of: (a) a line from A to an intermediate point C at the left of the curve; for moderate variations of T this line is virtually straight and its slope is as for the arrow starting from P in Figure VII-11; (b) a straight line going from C to B, with a slope as that of the straight line in Figure VII-6. **12.** (a) $R\Delta T\ (n - 1)$; (b) $-1.44 \times 10^4\ \mathrm{J\,kg^{-1}}$; $7.2 \times 10^3\ \mathrm{J\,kg^{-1}}$. **13.** $T = [T_0/(1 - \varkappa)]\,[(p_1^{1-\varkappa} - p_2^{1-\varkappa})/(p_1 - p_2)]p^\varkappa$. **14.** $T_\mathrm{aw} = 16.2\,^\circ\mathrm{C}$; $\theta_\mathrm{aw} = 20.2\,^\circ\mathrm{C}$; $T_\mathrm{s} = 13.0\,^\circ\mathrm{C}$; $p_\mathrm{s} = 827$ mb. **15.** (a) $\theta_\mathrm{ae} = \theta\exp(l_\mathrm{v}r/c_\mathrm{p}T_\mathrm{s}) = T(p_0/p)^\varkappa \exp(l_\mathrm{v}r/c_\mathrm{p}T_\mathrm{s})$ where $p_0 = 1000$ mb, $r = r_\mathrm{w}(T_\mathrm{s})$. Consider ascent from p, T first to saturation level p_s, T_s; then apply Equation (82) from $p_\mathrm{s}T_\mathrm{s}$ to $p'T'$ at a high enough level to consider $r'_\mathrm{w}(T') \cong 0$; then apply equation to the descent from $p'T'$ to p_0, $T = \theta_\mathrm{ae}$. **16.** $r_\mathrm{w} = 5.0\ \mathrm{g\,kg^{-1}}$; $\theta = 10.8\,^\circ\mathrm{C}$; $\theta_\mathrm{v} = 11.3\,^\circ C$; $p_\mathrm{s} = 810$ mb; $T_\mathrm{s} = -6.0\,^\circ\mathrm{C}$; $T_\mathrm{aw} = -0.8\,^\circ\mathrm{C}$; $\theta_\mathrm{aw} = 4.4\,^\circ\mathrm{C}$; $T_\mathrm{d} = -4.7\,^\circ\mathrm{C}$; $T_\mathrm{iw} = -0.4\,^\circ\mathrm{C}$; $T_\mathrm{ie} = 9.6\,^\circ\mathrm{C}$. **17.** It could be the air at 1000 mb; it could not be the air at 800 mb. **18.** θ_aw, θ_ae; T_d, q, r.

Chapter VIII

1. $18.8\ \mathrm{K\ gpkm^{-1}}$. **2.** (a) $p_0 = 1004.8$ mb. (b) Differentiate $p_0 = p(1 + \phi\gamma_\mathrm{v}/T_\mathrm{v})^{1/R_\mathrm{d}\gamma_\mathrm{v}}$ logarithmically, and make use of the approximations $\ln(1 + x) \cong x - x^2/2$ and $(1 + x)^{-1} \cong 1 - x$ for $x \ll 1$. The result is: $\mathrm{d}\ln p_0 \cong -(\phi^2/2R_\mathrm{d}T_\mathrm{v}^2)\,\mathrm{d}\gamma_\mathrm{v}$, defining the relative error in p_0. (c) $\mathrm{d}p_0 = 0.4$ mb. **3.** 1574 gpm. **4.** 842 gpm = 826 dyn-m = $8260\ \mathrm{J\,kg^{-1}}$. **5.** Differentiate $\phi = R_\mathrm{d}\bar{T}\ln(p_0/p_\mathrm{h})$ with respect to the horizontal distance x and obtain $\ln(p_0/p_\mathrm{h}) = -(\bar{T}/p_0)(\mathrm{d}p_0/\mathrm{d}x)/(\mathrm{d}\bar{T}/\mathrm{d}x) = 0.1197$. Introduce into ϕ and find $z = \phi/g = 953$ m.

Chapter IX

1. By introducing the equations of hydrostatic equilibrium $\mathrm{d}p = -(g/v_\mathrm{e})\,\mathrm{d}z$ and of adiabatic expansion $pv^n = \mathrm{const.}$ into $\delta a = -p\,\mathrm{d}v$. **2.** (a) Negative: $0.020\ \mathrm{cal\,g^{-1}}$; positive: $0.285\ \mathrm{cal\,g^{-1}}$. (b) Both 0. (c) 920 to 750 mb. (d) LCL: 845 mb; FCL: 700 mb. (e) $I = 5.2\,^\circ\mathrm{C}$. (f) CCL: 780 mb. **3.** (a) $r = 12.0\ \mathrm{g\,kg^{-1}}$; $U_\mathrm{w} = 67\%$; $\theta = 27.0\,^\circ\mathrm{C}$; $T_\mathrm{aw} = 17.9\,^\circ\mathrm{C}$; $\theta_\mathrm{aw} = 19.9\,^\circ\mathrm{C}$; $\theta_\mathrm{ae} = 62\,^\circ\mathrm{C}$; $T_\mathrm{iw} = 18.0\,^\circ\mathrm{C}$; $T_\mathrm{ie} = 52.5\,^\circ\mathrm{C}$. (b) LCL: 860 mb;

FCL: 633 mb; pseudolatent type. **4.** $\theta = 17.0\,^\circ\mathrm{C}$; $\theta_w = 15.0\,^\circ\mathrm{C}$; LCL: 954 mb; $T_s = = 13.1\,^\circ\mathrm{C}$; FCL: 870 mb; CCL: 910 mb; layer with latent instability: 1000 to 825 mb. **5.** (a) LCL: 945 mb; CCL: 835 mb. (b) Before: $T_d = 15.8\,^\circ\mathrm{C}$; $T_{aw} = \theta_{aw} = 17.2\,^\circ\mathrm{C}$. After: $T_d = 13.9\,^\circ\mathrm{C}$; $T_{aw} = \theta_{aw} = 18.2\,^\circ\mathrm{C}$. (c) $16.2\ \mathrm{m\,s^{-1}}$. (d) 1000 to 800 mb. **6.** (a) $\theta_{aw} = = 19.3\,^\circ\mathrm{C}$; $T_d = 16.7\,^\circ\mathrm{C}$; LCL = 890 mb; FCL = 830 mb. (b) real latent type. (c) 2324 gpm = $22791\ \mathrm{m^2\,s^{-2}}$. **7.** If the layer is saturated, it is unstable with respect to parcel convection. If it is not saturated, it will be stable or unstable with respect to parcel convection, according to whether $\delta\theta/\delta z \gtrless 0$, and it is potentially unstable (regarding layer instability). **8.** (a) $\delta\theta_v/\delta\phi = 2.6\ \mathrm{K\ gpkm^{-1}} < \mathrm{d}\theta_v/\mathrm{d}\phi = 2.8\ \mathrm{K\ gpkm^{-1}}$: unstable. (b) stable. **9.** (a) $n = 1/(1 - R_d\beta/g) = 0.72$ ($\beta = -0.0134\ \mathrm{K\,m^{-1}}$). (b) $\delta q/\mathrm{d}z = g - c_p\beta = = 5.6\ \mathrm{cal\,kg^{-1}}$. **10.** Parcel lapse rate: $\gamma' = \gamma_d(1 - \delta q/\mathrm{d}z)$. (a) $\gamma' = 1.4\ \mathrm{K\ gpkm^{-1}} < 5\ \mathrm{K\ gpkm^{-1}}$: unstable. (b) $\gamma' = 5.6\ \mathrm{K\ gpkm^{-1}}$: stable. **11.** 1.72 km. **12.** $\delta q/\mathrm{d}z = c_p(T/\theta)\,\mathrm{d}\theta/\mathrm{d}z$; $240\ \mathrm{cal\,kg^{-1}}$. **13.** $5.2\ \mathrm{mm\,h^{-1}}$. **14.** $U = 1.9 \times 10^9\ \mathrm{J\,m^{-2}}$; $P = 0.77 \times 10^9\ \mathrm{J\,m^{-2}}$; $H = 2.7 \times 10^9\ \mathrm{J\,m^{-2}}$. **15.** (a) 1.73×10^3 gpm. (b) $\Delta h = 1.7 \times 10^4\ \mathrm{J\,kg^{-1}}$; $\Delta u = 1.2 \times 10^4\ \mathrm{J\,kg^{-1}}$; $\Delta s = 0$. (c) $U = 1.94 \times 10^8\ \mathrm{J\,m^{-2}}$. **16.** (a) $\Delta\phi_i = = 1706$ gpm; $\Delta\phi_f = 1773$ gpm. (b) 67 gpm. (c) $\Delta U = 1.56 \times 10^7\ \mathrm{J\,m^{-2}}$; $\Delta P = = 8.4 \times 10^5\ \mathrm{J\,m^{-2}}$; $\Delta H = 2.18 \times 10^7\ \mathrm{J\,m^{-2}}$.

18. From Equations (169) and (177),

$$\frac{p'}{p} = s\left(1 - \frac{2x}{L}\right)$$

From Chapter VIII, Equation (39) and expressing the temperatures in terms of potential temperature, we find for a constant lapse rate column

$$\frac{p}{p_0} = \left(\frac{\theta}{\theta_0}\right)^{-1/(\varkappa - R\gamma)}$$

In the present example, for $\theta > \theta_0$,

$$\bar{p}(\theta) = p_0\left(\frac{\theta}{\theta_0}\right)^{-1/(\varkappa - R\gamma)}$$

where $p_0 = p(L/2, \theta_0)$ and γ is the lapse rate at $x = L/2$. For $\theta < \theta_0$, $\bar{p}(\theta) = p_0$, as explained in the text. Applying Equation (172), we obtain

$$A = WL\frac{c_p\varkappa(1 + \varkappa)s^2}{6gp_{00}^{\varkappa}(1 + R\gamma)}\,p_0^{1+\varkappa}\theta_0$$

The total enthalpy H is calculated from Equation (149) with $p_h = 0$ and $H = \eta U$:

$$H = WL\frac{c_p\theta_0 p_0^{1+\varkappa}}{g(1 + R\gamma)p_{00}^{\varkappa}}$$

The values of A and H referred to unit area are obtained by dividing by WL Introducing the values of p_0, θ_0, γ and s given in the text the numerical results mentioned are obtained.

INDEX OF SUBJECTS