CHAPTER VI

AEROLOGICAL DIAGRAMS

6.1. Purpose of Aerological Diagrams and Selection of Coordinates

In order to be able to study with speed and convenience the vertical structure and a number of properties of the atmosphere above a certain location, use is made of special thermodynamic diagrams, or *aerological diagrams*. The observational data to be represented are obtained from soundings, and consist of sets of values of temperature, pressure, and humidity (e.g., relative humidity).

These data are considered as essentially vertical and instantaneous. With a net of sounding stations, they can be organized in a tridimensional description of the atmosphere, and with successive soundings, evolution with time can be followed.

The system to be represented on these diagrams will be in general moist air. It has two components and one phase; its variance, according to the phase rule, must be three, corresponding to the three different variables measured in the soundings. As there are three independent variables, p, T and U_w (or U_i) that have to be plotted on a plane surface, the representation should consist of one curve with a set of scalars (e.g., p as a function of T, and values of U_w written for points along the curve), or two different curves (e.g., T as a function of p and U_w as a function of p). If there is water (or ice) present, as in a cloud, the variance is reduced to two, and one curve is enough (e.g., T as a function of p; $e=e_w(T, p)$ is not independent).

On a thermodynamic diagram, different *isolines* or *isopleths* can be drawn: isobars, isotherms, equisaturated curves or 'vapor lines' (curves for which $U_w=1$), curves of equal potential temperature or isentropics (cf. Chapter III, Section 9) for dry air, or 'dry adiabats', etc. The ones specifically mentioned and the 'saturated adiabats', to be studied later, are the most important, and in that sense we shall call them the *fundamental lines* of the diagram. It should be remarked that these lines, as any other curve drawn on the diagram, may have two meanings. One, static, indicating the vertical structure of an atmospheric layer – thus, an adiabat may represent an atmosphere with constant value of the potential temperature (and of the specific entropy) along the vertical. But it may also have the meaning of a *process curve*, representing the change of the variables for an air parcel undergoing an adiabatic expansion as it rises through the atmosphere. The coincidence of representation means that the potential temperature is an invariant for the process just mentioned.

The importance of the diagrams lies in the large amount of information that can be rapidly obtained from them. They allow the study of the vertical stability of the atmosphere. The thickness of layers between two given values of the pressure is readily computed from them. A number of atmospheric processes can be conveniently studied with their aid. The vertical structure will also indicate the type of air mass or air masses involved.

These uses determine the choice of the coordinates, and many different diagrams have been developed for general and for special purposes. We shall only see a few of the most commonly used. There are certain criteria which are taken into account in the selection of an appropriate diagram; we enumerate them below.

(a) The angle between isotherms and adiabats. The larger this angle is, the more sensitive is the diagram to variations in the rate of change of temperature with pressure along the vertical. This is important, as we shall see, for the analysis of stability, including synoptic aspects of frontal and air mass analysis.

(b) Which and how many isopleths are straight lines. Straight lines facilitate the use of the diagram for plotting as well as for analysis and representation.

(c) If energy integrals (such as, for instance, the work performed by a parcel of moist air along a cycle) can be determined by measuring areas on the diagram. Such diagrams are sometimes called *equivalent*, or *area-preserving*.

(d) Some advantage may be obtained if one of the main isopleths is congruent with respect to a displacement along one of the coordinates (cf. Sections 3 and 5).

(e) If the ordinate varies monotonically with height, roughly proportional to it, the atmosphere can be more conveniently imagined or visualized over the diagram.

6.2. Clapeyron Diagram

Clapeyron diagram uses the coordinates $-p$, v*. It is the diagram of frequent use in general thermodynamics, with a changed sign for p, to comply with condition (e) of Section 1. As for the other conditions (cf., Figure VI-1):

(a) The angles between adiabats and isotherms are small.

(b) Isobars and isochores are straight lines. Adiabats and isotherms are curves.

(c) It is equivalent; in a cycle: $a = \oint (-p)\, \mathrm{d}v$.

This is not a convenient diagram and therefore it is not used. We mention it here only for comparison purposes, as it is a well-known representation in thermodynamics.

6.3. Tephigram

The coordinates are $\ln\theta$, T. Its name, given by Shaw, comes from the letters T and ϕ, the latter as a symbol of entropy. As we saw in Chapter III, Section 9, the ordinate variable $\ln\theta$ is proportional to the specific entropy, so that this diagram can also be considered as having coordinates s, T.

Although the ordinates are proportional to $\ln\theta$, the adiabats (straight lines of constant θ) are labelled with the values of θ; i.e., the vertical axis is a logarithmic scale of θ. The diagram is represented schematically in Figure VI-2.

* We shall mention first the ordinate and then the abscissa. This convention is opposite to the general use in mathematics, but is frequently applied in thermodynamics.

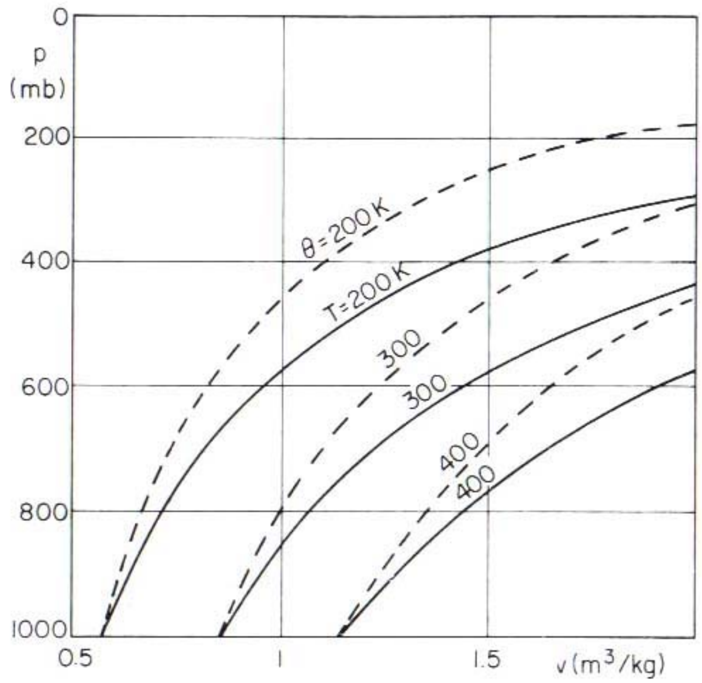

Fig. VI-1. Clapeyron diagram.

As for properties of the tephigram:

(a) The angle between adiabats and isotherms is equal to 90°, a very favorable condition.

(b) Isotherms and adiabats are straight lines. From the equation of adiabats

$$\theta = T\left(\frac{1000}{p}\right)^{\varkappa_d} \tag{1}$$

(where p is expressed in mb), we have

$$(\ln \theta) = \ln (T) - \varkappa_d \ln p + \text{const.} \tag{2}$$

where the brackets indicate which are the variables of the diagram. Isobars (p=constant) are therefore logarithmic, but their curvature is small within the usual range.

(c) It is equivalent. This can be shown by considering the work a performed on a unit mass of air in a reversible cycle

$$a = -q = -\oint T\,\mathrm{d}s. \tag{3}$$

Introducing (Chapter III, Equation (54)): $\mathrm{d}s = c_p\,\mathrm{d}\ln\theta$, we obtain:

$$a = -c_p \oint T\,\mathrm{d}\ln\theta = -c_p \textstyle\sum_{\text{teph}} \tag{4}$$

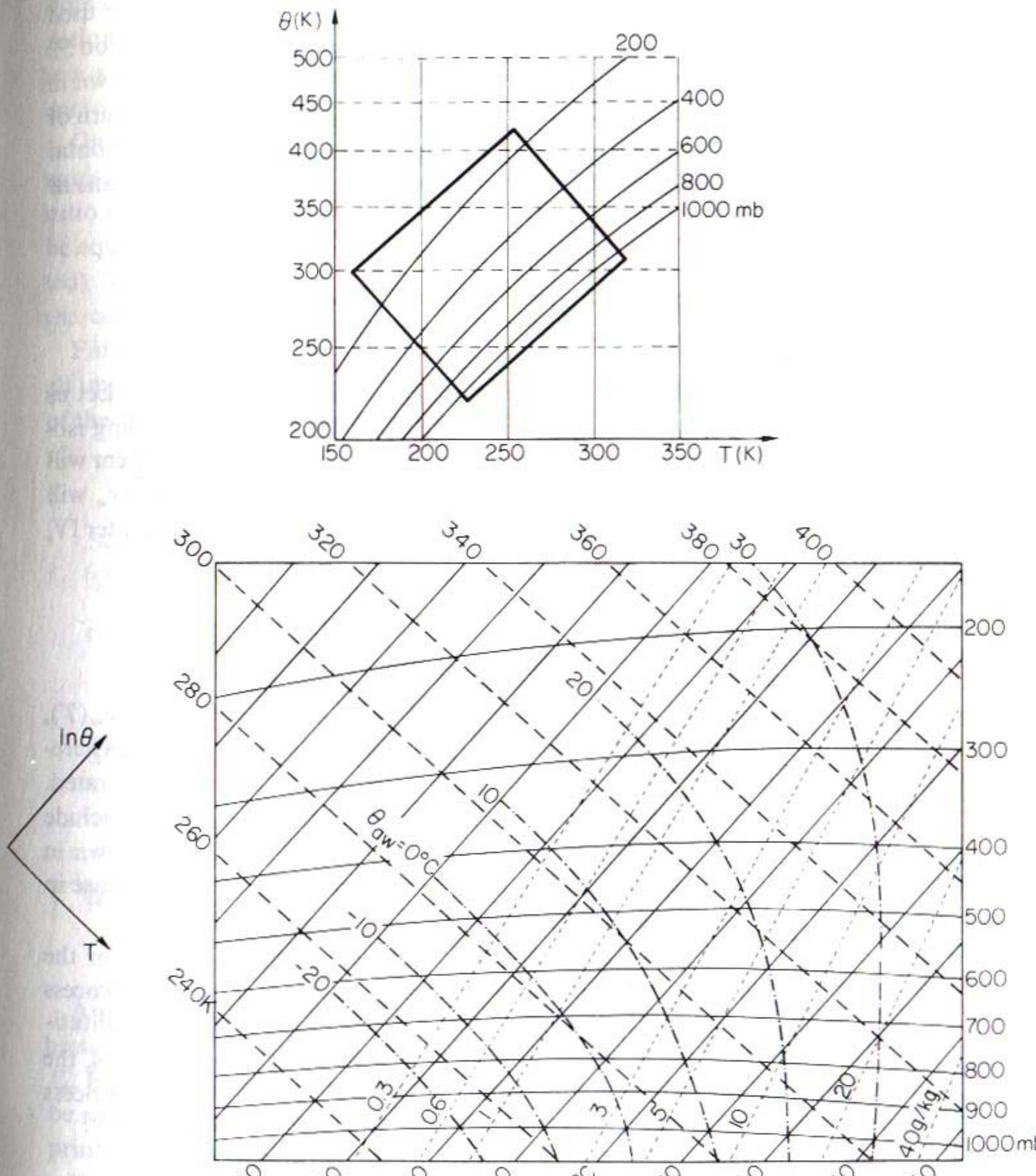

Fig. VI-2. Tephigram.

where $\sum_{\rm teph}$ is the area determined by the cycle on the diagram, taken as positive if described counterclockwise, as the integration has been performed taking the vertical coordinate as the independent variable. If the air is dry, $c_p = c_{p_d}$.

(d) From Equation (2) it is obvious that isobars are congruent with respect to a displacement along the ordinate. Thus the isobar kp may be obtained by displacing vertically the isobar p by the constant quantity $\varkappa_d \ln k$. This property could be used to represent two different intervals of p and θ with the same set of curves (see Section 5).

(e) To comply with this criterion, the zone of interest in the atmosphere (shown in a rectangle at the top of Figure VI-2) is frequently represented, after a clockwise turn of about 45°, as in the lower part of Figure VI-2. Thus isobars approximate horizontal straight lines. We shall frequently use this arrangement in our schematic diagrams of the following chapters.

6.4. Curves for Saturated Adiabatic Expansion. Relative Orientation of Fundamental Lines

We consider now the isotherms and the equisaturated lines on the tephigram. Let us assume that, starting from an image point P, we go up along the corresponding isotherm T to point P', which has the pressure $p' < p$ (see Figure VI-3). Our system will consist of a certain mass of saturated air, and will be closed. Its mixing ratio r_w will therefore remain constant, unless condensation occurs. The first formula in Chapter IV, Equation (79)

$$r_w = \frac{\varepsilon e_w(T)}{p - e_w(T)}$$

shows that the saturation mixing ratio at the new pressure, r'_w, exceeds r_w, as $e_w(T)$, which is only a function of T, remains constant, while the pressure p is lower. Therefore, there was no condensation and, on the contrary, the air has become unsaturated. The saturation mixing ratio also increases if we increase T at constant p. We conclude that the equisaturated line must be inclined to the left of the isotherm, as shown in Figure VI-3, in such a way that the decrease in $e_w(T)$ must compensate the decrease in p (in order to keep a constant mixing ratio).

Let us consider now a new process, the adiabatic ascent of saturated air, for the sole purpose of describing the diagrams. Analytical consideration of this process will be given in Chapter VII, Sections 8 and 9; here we shall only consider it qualitatively. The ascent of the air implies an expansion, and being an adiabatic process, the temperature decreases and water vapor condenses. The curve describing this process

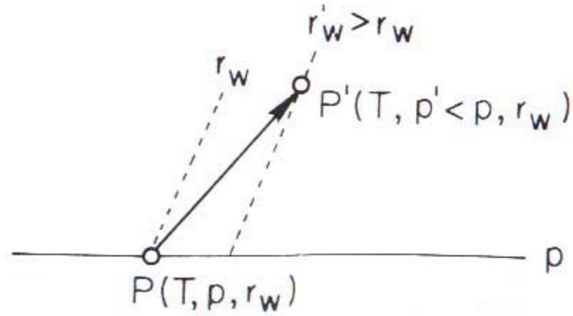

Fig. VI-3. Relative orientation of isotherms and equisaturated lines.

does not follow the non-saturated adiabat any more, because the latent heat of vaporization released during the condensation is absorbed by the air, with the effect that there is a smaller drop in temperature for the same decrease in pressure. Therefore this curve, when considered from point P up, is situated to the right of the dry adiabat.

On the other hand, the saturated adiabat must cut, in the direction of expansion, equisaturated lines of lower and lower values of r_w, in view of the fact that the mixing ratio of the air diminishes continuously with condensation, and in every point must be equal to that of the equisaturated line passing through it (because the air is saturated). This requires that the inclination of the saturated adiabat should be such that the curve lies, for every point, between the dry adiabat and the equisaturated line.

Finally, it is obvious that an isobaric cooling must be beyond all these curves, which all imply a decrease in pressure. We conclude, therefore, that the relative orientation of the fundamental lines must be that indicated in Figure VI-4.

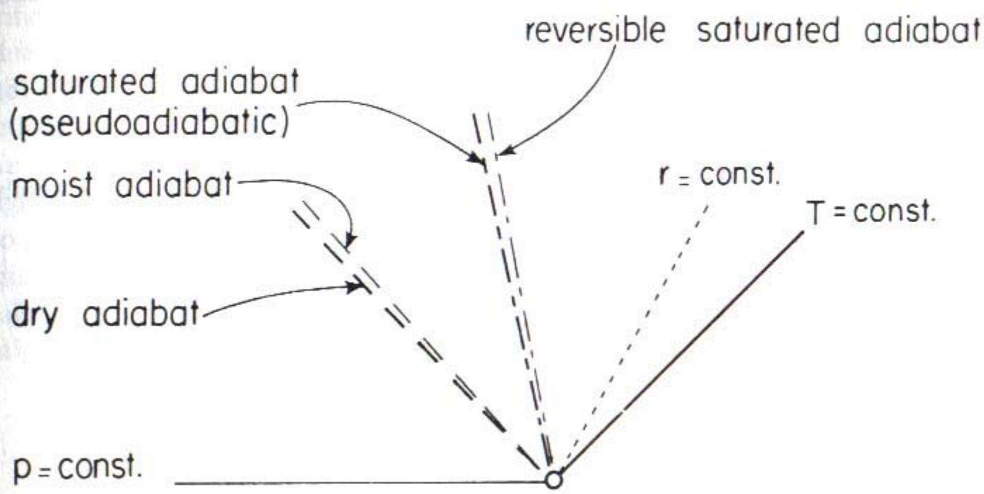

Fig. VI-4. Relative orientation of fundamental lines.

An adiabat corresponding to moist air will show a variation of temperature with pressure slightly smaller than that for dry air (because $\varkappa < \varkappa_d$; see Chapter IV, Section 12, Equation (89)), and its slope will depend on the specific humidity; its orientation will be such as indicated with a thin dashed curve in Figure VI-4. Only dry adiabats are printed on diagrams.

We must also remark that the saturated adiabats printed on the diagrams correspond to the process in which all the water goes out of the system as soon as it is condensed (pseudo-adiabatic process; Chapter VII, Section 9). If we draw the curves corresponding to the process in which all the water remains in the system (reversible saturated adiabatic process; Chapter VII, Section 8), they should show a slightly smaller slope, because the liquid water will also lose heat while cooling. This is shown in Figure VI-4 by the thin dash-and-dot curve.

This sequence in the orientation of the fundamental lines holds for all diagrams. As in the previous figures, we shall always use the following convention to represent the fundamental lines: full lines for isotherms and isobars, dashed curves for dry adiabats, dash-and-dot curves for saturated adiabats and dotted (or short-dashed) curves for equisaturated lines. Values of pressures are given in mb, and values of mixing ratios in g kg^{-1}.

6.5. Emagram or Neuhoff Diagram

We consider now the diagram with coordinates $-\ln p$ (logarithmic scale of p), T (Figure VI-5). It is usually called the *emagram*, following Refsdal, from '*e*nergy per unit *ma*ss dia*gram*', although this name could also be applied to all equivalent diagrams in general. Regarding its main properties:

(a) The angles depend on the scales used for the coordinate axes and vary with the point; the scales are usually chosen so that they are about 45°.

(b) Isobars and isotherms are straight lines. Adiabats can be written (cf. Equation (1)) as

$$(-\ln p) = -\frac{1}{\varkappa_d}\ln(T) + \frac{1}{\varkappa_d}\ln\theta + \text{const.} = -\frac{1}{\varkappa_d}\ln(T) + \text{const.} \qquad (5)$$

where the two coordinate variables are between brackets; this shows that adiabats are logarithmic curves (normally with a small curvature).

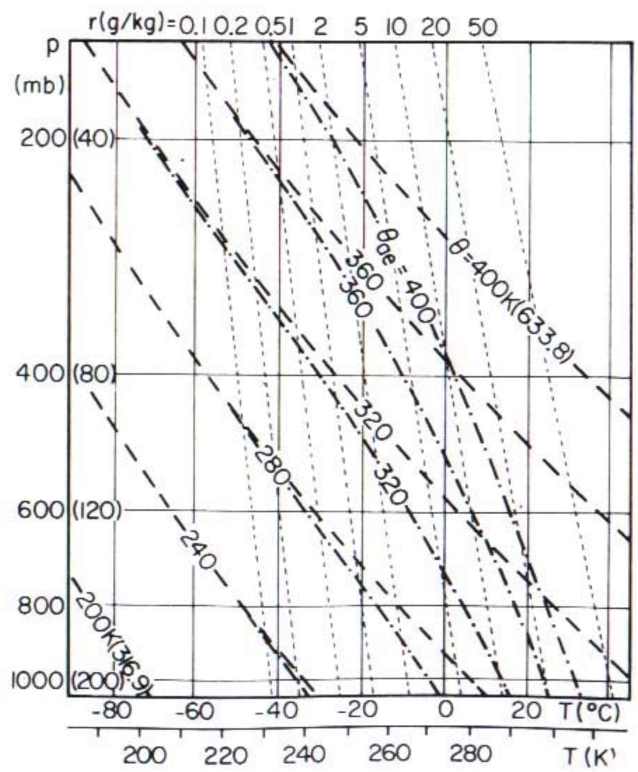

Fig. VI-5. Emagram.

(c) It may be shown that it is an equivalent diagram. We have:

$$\delta a = -\, p\, \mathrm{d}v = -\, d(pv) + v\, \mathrm{d}p = -\, R\, \mathrm{d}T + v\, \mathrm{d}p\,.$$

In a cycle

$$a = -\, R \oint \mathrm{d}T + \oint v\, \mathrm{d}p = \oint v\, \mathrm{d}p = -\, R \oint T\, \mathrm{d}(-\ln p) = -\, R \textstyle\sum_{\mathrm{em}} \quad (6)$$

where $\sum_{\mathrm{em}}$ stands for the area enclosed in the cycle, positive if described counterclockwise. For dry air, $R = R_{\mathrm{d}}$.

(d) Adiabats are congruent with respect to a displacement along the vertical axis. This can be seen from Equation (5), taking into account that $\ln(kp) = \ln p + \ln k = \ln p + {}$ $+ \text{const}$. This property can be used to represent two different intervals of pressures and potential temperatures with the same set of curves. Thus the same curve represents p as a function of T for $\theta = \text{const.}$ and (kp) as a function of T for $(k^{-\varkappa_d}\theta)$, as can be verified by adding $-\ln k$ on both sides of Equation (5); this is done in Figure VI-5 for a value $k = 1/5$, with some values of (kp) and $(k^{-\kappa_d}\theta)$ indicated between brackets.

(e) Ordinates are roughly proportional to heights, as will be seen in Chapter VIII, Section 11.

In order to increase the angle between isotherms and adiabats, thereby making the diagram more sensitive to changes of slope of the curves $T = \mathrm{f}(p)$, a *skew emagram* is also used with the axes at an angle of 45°, as indicated in Figure VI-6. Comparison with Figure VI-2 shows that the skew emagram is very similar to the tephigram, with straight isobars and slightly-curved adiabats replacing slightly-curved isobars and straight adiabats.

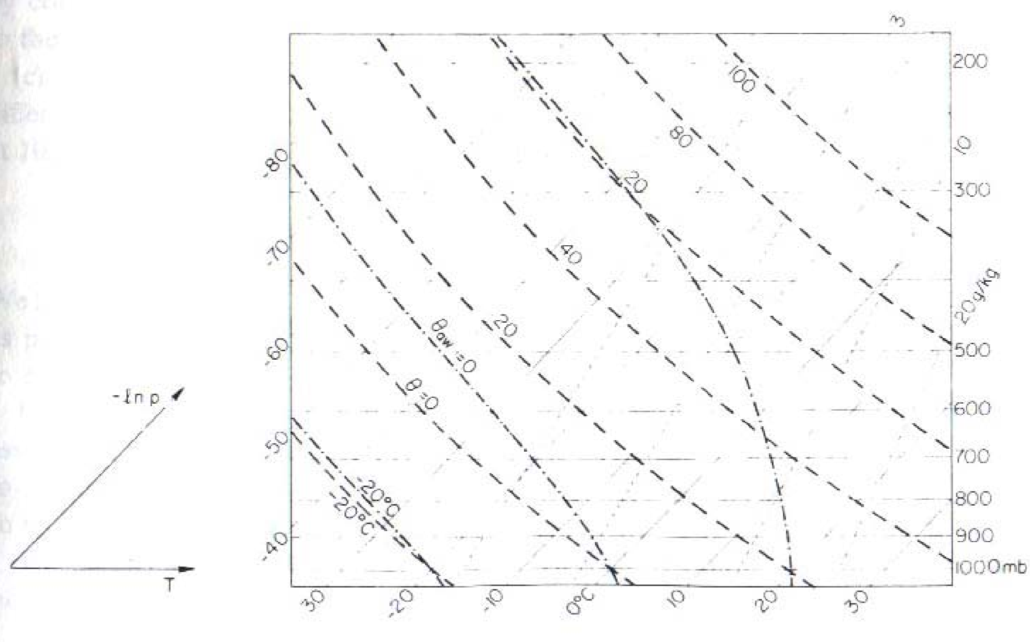

Fig. VI-6. Skew-emagram.

6.6. Refsdal Diagram

The diagram of Refsdal has coordinates $-T\ln p$, $\ln T$, labelled with the values of p and T, respectively. Regarding some of its properties:

(a) Only isotherms are straight lines.

(b) The angle between adiabats and isotherms depends on the scales.

(c) It is equivalent. Integrating with respect to the abscissa:

$$\int (-T\ln p)\,\mathrm{d}\ln T = -\int \ln p\,\mathrm{d}T = -T\ln p + \int T\,\mathrm{d}\ln p\,. \tag{7}$$

In a cycle the first term cancels and the area becomes (changing sign for a cycle described counterclockwise):

$$\textstyle\sum_{\mathrm{Ref}} = -\oint T\,\mathrm{d}\ln p = \oint T\,\mathrm{d}(-\ln p)\,. \tag{8}$$

Therefore, for a cycle (cf. Equation (6)):

$$a = -R_{\mathrm{d}}\textstyle\sum_{\mathrm{Ref}}\,. \tag{9}$$

(d) Neither adiabats nor isobars are congruent for displacement along the axis.

(e) As will become clear in Chapter VIII, Section 11, the vertical coordinate is essentially proportional to height in the atmosphere.

Figure VI-7 represents a simplified Refsdal diagram.

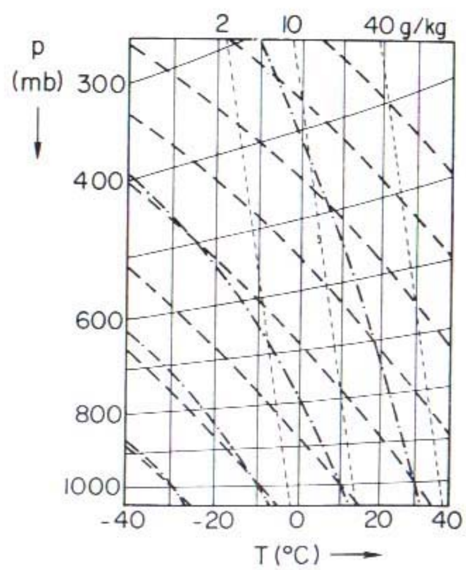

Fig. VI-7. Refsdal diagram.

6.7. Pseudoadiabatic or Stüve Diagram

The pseudoadiabatic or Stüve diagram, shown schematically in Figure VI-8, has the coordinates $-p^{\varkappa_d}$, T, the ordinates being labelled with the values of p. The region of the appropriate range of the variables which is usually represented corresponds to the rectangle in thick lines shown in the figure.

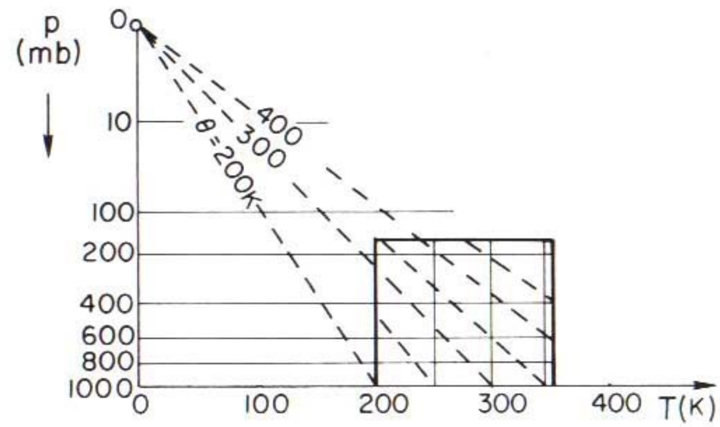

Fig. VI-8. Pseudoadiabatic or Stüve diagram.

Some of its properties are:

(a) The angle between adiabats and isotherms depends on the scales and is normally around 45°.

(b) Isotherms and isobars are straight lines, as well as adiabats, as becomes obvious by considering Poisson's equations (Chapter II, Section 7). The adiabats converge to the point $p=0$, $T=0$, as shown in the figure.

(c) It is not equivalent. However, energies corresponding to equal areas do not differ greatly; for instance, 1 cm^2 at 400 mb represents about 25% more energy than at 1000 mb.

6.8. Area Equivalence

We have seen already the area-energy equivalence properties of the diagrams described, as particular cases. Here we shall describe a general method which may be applied to check that property on any diagram.

Let us call $\sum$ the area enclosed by a certain contour C in the x, y plane. We shall assume that these two variables are related to another pair u, w, that there is a one-to-one correspondence between the points in the plane x, y and those in u, w, and that to the points in $\sum$ correspond those of a surface $\sum'$ enclosed by a contour C'.

The area of an elementary surface $\mathrm{d}\sum$ in x, y may be expressed by the cross product of two vectors $\mathbf{dx}$ and $\mathbf{dy}$:

$$\mathrm{d}\sum = \mathbf{dx} \times \mathbf{dy}.$$

Similarly

$$\mathrm{d}\sum{}' = \mathbf{du} \times \mathbf{dw}$$

(see Figure V-9).

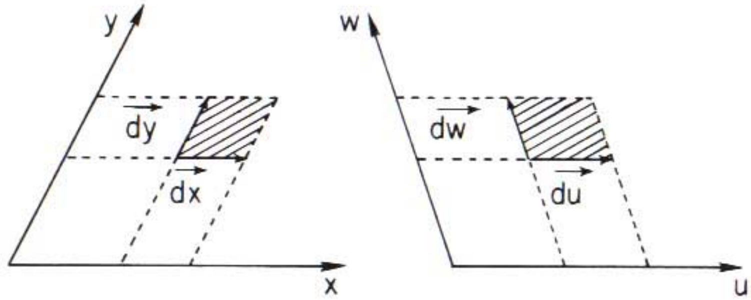

Fig. VI-9. Area equivalence.

Every point can be represented in each plane by two vectors; in the x, y plane: $x = x(u, w)$, $y = y(u, w)$, where the functions represent the relation between the points in both planes. Differentiation gives:

$$\mathbf{dx} = \frac{\partial x}{\partial u}\,\mathbf{du} + \frac{\partial x}{\partial w}\,\mathbf{dw}$$

$$\mathbf{dy} = \frac{\partial y}{\partial u}\,\mathbf{du} + \frac{\partial y}{\partial w}\,\mathbf{dw}$$

and

$$\mathrm{d}\sum = \left(\frac{\partial x}{\partial u}\,\mathbf{du} + \frac{\partial x}{\partial w}\,\mathbf{dw}\right) \times \left(\frac{\partial y}{\partial u}\,\mathbf{du} + \frac{\partial y}{\partial w}\,\mathbf{dw}\right)$$

$$= \left(\frac{\partial x}{\partial u}\frac{\partial y}{\partial w} - \frac{\partial x}{\partial w}\frac{\partial y}{\partial u}\right)(\mathbf{du} \times \mathbf{dw})$$

where we have taken into account that $\mathbf{du} \times \mathbf{du} = \mathbf{dw} \times \mathbf{dw} = 0$ and $\mathbf{du} \times \mathbf{dw} = -\mathbf{dw} \times \mathbf{du}$. Therefore

$$\mathrm{d}\sum = J\,\mathrm{d}\sum{}' \tag{10}$$

where

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial w} \\ \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial w} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial w} - \frac{\partial x}{\partial w}\frac{\partial y}{\partial u} = J\frac{(x, y)}{(u, w)}.$$

J is called the Jacobian of the co-ordinate transformation.

The condition for the transformation to be area-preserving is that the Jacobian be equal to unity or to a constant. In the last case the area in the new co-ordinates will be proportional to that in the first system. If in the case of aerological diagrams, $\sum$ is proportional to the energy, $\sum'$ will also be proportional to it, but with a different proportionality constant, as shown by Equation (10), integrated to

$$\sum = J\sum'. \tag{11}$$

In order to see if a diagram is area-equivalent, it will be sufficient to calculate the Jacobian that relates it to Clapeyron's diagram (or to any other area-equivalent diagram). The work performed on the system per unit mass in a process will be proportional to the area determined by the process if J is constant, and J will be the proportionality constant.

As an example, we shall demonstrate the area equivalence of the emagram, starting from Clapeyron's diagram.

We have to transform the coordinate system form $x = v$, $y = -p$ to $u = T$, $w = -\ln p$.

$$x = v = \frac{RT}{p} = Rue^{w}$$

$$y = -p = -e^{-w}$$

$$\frac{\partial x}{\partial u} = Re^{w} \qquad \frac{\partial x}{\partial w} = Rue^{w}$$

$$\frac{\partial y}{\partial u} = 0 \qquad \frac{\partial y}{\partial w} = e^{-w}$$

$$J = R. \tag{12}$$

If we take into account that the integrations are performed in such a way that the area is described clockwise in the Clapeyron diagram and counterclockwise in the emagrams, we must add a negative sign. As the area in the Clapeyron diagram gives the work a, this result, introduced into Equation (11), reproduces formula (6).

6.9. Summary of Diagrams

The main properties of the diagrams mentioned are summarized in the following table.

Diagram	Abscissa	Ordinate	J (from $-p, v$)	Straight lines			Angle adiabats-isotherms
				Isobars	Adiabats	Isotherms	
Clapeyron	v	$-p$	1	yes	no	no	small
Tephigram	T	$\ln\theta$	c_p	no	yes	yes	90°
Emagram	T	$-\ln p$	R	yes	no	yes	~45° (unless skewed)
Refsdal	$\ln T$	$-T\ln p$	R	no	no	yes	~45°
Pseudo-adiabatic or Stüve	T	$-p^{\varkappa_d}$	$\neq$ const	yes	yes	yes	~45°

6.10. Determination of Mixing Ratio from the Relative Humidity

In Chapter IV, Equations (85) and (86) we have seen the expressions to compute r from U_w, which is generally the parameter directly obtained from observations. A simple rule (Refsdal) allows one to determine it conveniently on the diagrams. The procedure is:

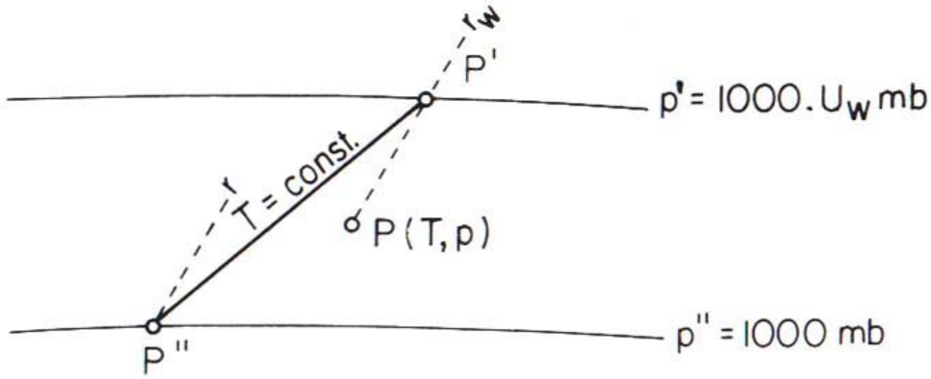

Fig. VI-10. Determination of mixing ratio from the relative humidity.

(1) From the image point $P(T, p)$ (see Figure VI-10) follow the vapor line passing through it up to its intersection P' with the isobar $p = 1000\ U_w$ mb.

(2) Follow the isotherm passing through P' down to its intersection with the 1000 mb isobar (P''). The value of the vapor line passing through P'' gives the mixing ratio r of the air.

This may be demonstrated using the approximate formulae of Chapter IV, Section 11:

$$r = \frac{\varepsilon e_w(T')}{p''} = \frac{\varepsilon U_w e_w(T')}{p'} = U_w r_w(T', p') = U_w r_w(T, p) \tag{13}$$

where the first equation comes from applying the general formula $r_w \cong \varepsilon e_w/p$ to P'' (r being the saturation value at that point), the second expression is obtained by substituting $p'' = p'/U_w$, and the third expression from applying $r \cong \varepsilon e_w/p$ to P'.

6.11. Area Computation and Energy Integrals

We have seen that in area-equivalent diagrams, the area enclosed by a cyclic process is proportional to the work a and to the heat $q = -a$ received by unit mass of air during the process. We are also interested in the values of these quantities for any open processes. We shall consider such processes for moist air, when no condensation takes place. In that case, the variation in state functions will no longer be zero in general. We shall have:

$$\Delta u = c_v \Delta T \tag{14}$$

$$\Delta h = c_p \Delta T \tag{15}$$

$$\Delta s = c_p \ln \frac{\theta_2}{\theta_1} \tag{16}$$

where the subscripts 1, 2 refer to the initial and final states. ΔT can be read directly. Δs can be computed from values of θ_1 and θ_2 or by measuring $\Delta \ln\theta = \ln(\theta_2/\theta_1)$ on a tephigram with an appropriate scale in entropy units.

The calculation of the heat q and the work a, which are not state functions, requires a knowledge of the process undergone by the system. If this process is plotted on a diagram, the calculation can be simplified by graphical approximations. Let us consider the following expressions:

$$a = -\int_1^2 p\,\mathrm{d}v = -pv|_1^2 + \int_1^2 v\,\mathrm{d}p = -R\Delta T + R\int_1^2 T\,\mathrm{d}\ln p \tag{17}$$

$$q = \int_1^2 T\,\mathrm{d}s = c_p \int_1^2 T\,\mathrm{d}\ln\theta . \tag{18}$$

If we can calculate either q or a, the other one follows immediately from the first principle

$$\Delta u = a + q \tag{19}$$

and Equation (14).

The integral term in the last expression of a in Equation (17) is directly related to an emagram area. If we call $\sum_{\mathrm{em}}$ the area 1-2-3-4-1 in Figure VI-11 (described counter-clockwise), where the curve 1-2 represents the process, we have

$$\textstyle\sum_{\mathrm{em}} = \displaystyle\int_1^2 T\,\mathrm{d}(-\ln p) = -\int_1^2 T\,\mathrm{d}\ln p \tag{20}$$

so that

$$a = -R(\Delta T + \textstyle\sum_{\mathrm{em}}) \tag{21}$$

and (from Equations (14) and (19))

$$q = c_p \Delta T + R \textstyle\sum_{\mathrm{em}} . \tag{22}$$

It is obvious that $\sum_{\mathrm{em}}$ can be computed by drawing the mean temperature $\bar{T}$ that compensates areas (1′-2′ in the figure) and taking the area of the rectangle 1′-2′-3-4-1′. This is the *method of the mean isotherm*. It will be

$$\textstyle\sum_{\mathrm{em}} = -\bar{T} \ln \dfrac{p_2}{p_1} . \tag{23}$$

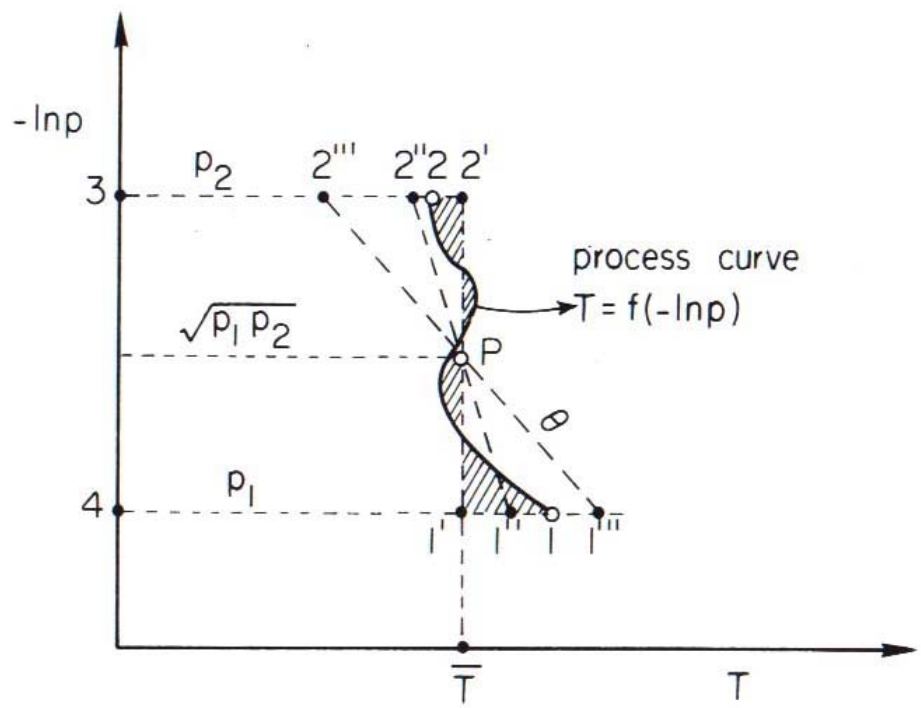

Fig. VI-11. Graphical computation of energy integrals on an emagram.

If we now substitute any straight line, such as 1″-2″, passing through the middle point P in the isotherm, for 1′-2′, this will determine the same area. Therefore, reversing the argument, we could choose any straight line following as closely as possible the curve 1-2 and compensating areas (which will be much easier to do with a good approximation); the middle point P of this straight line between the isobars will give $\bar{T}$. This is illustrated in Figure VI-11 by the line 1″-2″; it will be $\bar{T} = (T_1'' + T_2'')/2$, where T_1'' and T_2'' are the temperatures corresponding to the points 1″ and 2″. We notice that the isobar passing through P has the value $(p_1 p_2)^{1/2}$.

The area $\sum_{\text{em}}$ can also be computed by the *method of the mean adiabat*. This consists in finding the dry adiabat 1‴-2‴ that passes through 1-2, compensating areas. If p_1 and p_2 are not very distant, 1‴-2‴ can be taken on an emagram as a straight line, in which case it should pass through P. Now $\sum_{\text{em}}$ can be computed from the area 1‴-2‴-3-4-1‴; this gives

$$\sum_{\text{em}} = -\int_1^2 T \,\mathrm{d} \ln p = -\frac{1}{R}\int_1^2 v \,\mathrm{d}p = -\frac{1}{R}\int_{1'''}^{2'''} v \,\mathrm{d}p \tag{24}$$

where the last integral is taken along 1‴-2‴, i.e., corresponds to an adiabatic process for which $\delta q = 0$. Therefore, from the first principle:

$$\int_1^2 v \,\mathrm{d}p = \Delta h - q = \Delta h = c_\mathrm{p}(T_2'''-T_1''') \tag{25}$$

and

$$\sum_{em} = \frac{1}{\varkappa}(T_1'''\text{-}T_2''') \tag{26}$$

where T_1''' and T_2''' are the temperatures at which the area-compensating adiabat intersects the isobars p_1 and p_2.

Wherever the product RT appears in the formulas, it can be substituted by the equivalent $R_d T_v$ (Chapter IV, Section 11), coupling the use of the dry air constant R_d with that of the virtual temperature; thus Equation (21) can be written with R_d, ΔT_v and $\sum_{em}$ given by Equation (23) with $\bar{T}_v$ rather than $\bar{T}$. On the other hand, $c_p T \neq c_{p_d} T_v$, so that a similar substitution could not be made in Equation (25).

Equations (23) and (26) provide convenient methods for computing energy integrals on an emagram. Now we shall consider the use of the tephigram. The heat q can be expressed by

$$q = \int_1^2 T\,\mathrm{d}s = c_p \int_1^2 T\,\mathrm{d}\ln\theta\,. \tag{27}$$

The last integral is clearly related to the tephigram. If we consider Figure VI-12, and call $\sum_{te}$ the area 1-2-3-4-1 (1-2, as before, represents the process),

$$\sum_{te} = \int_1^2 T\,\mathrm{d}\ln\theta\,. \tag{28}$$

As for the emagram, we can apply the method of the mean isotherm (1′-2′ in the figure) and write

$$\sum_{te} = \bar{T}\ln\frac{\theta_2}{\theta_1} = \varkappa\bar{T}\ln\frac{p_1'}{p_2'} = \frac{1}{c_p}\bar{T}\Delta s \tag{29}$$

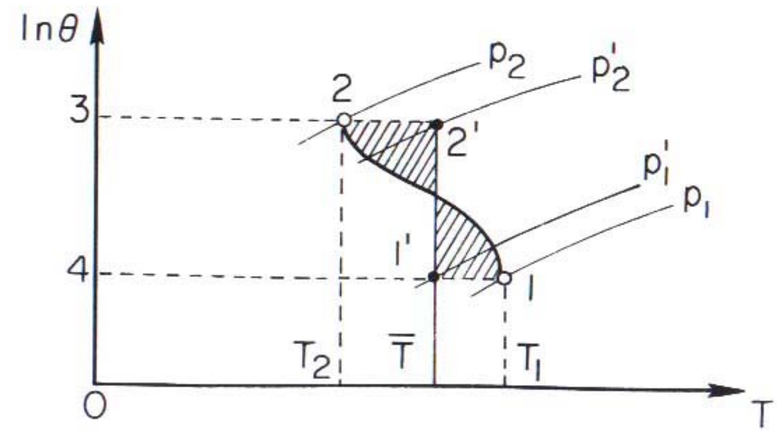

Fig. VI-12. Graphical computation of energy integrals on a tephigram.

where we have used Poisson's equation $\ln\theta = \ln T - \varkappa \ln p + \varkappa \ln 1000$ to relate the potential temperatures and the pressures along the isotherm $\bar{T}$, and the last expression indicates that a scale of entropies (rather than the logarithmic scale of θ) would provide a simple way of measuring the area.

It may be more convenient to work with the tephigram in a similar way as with the emagram – that is, integrating between isobars instead of between adiabats. We shall consider this procedure, which is not rigorous, but gives a good approximation provided the process occurs within a layer Δp which is not too thick.

We first consider that the pressure scale is logarithmic along an isotherm; for, by definition of θ,

$$\ln\theta = C - \varkappa \ln p \tag{30}$$

where C is a constant at constant T, and as $\ln\theta$ is the ordinate variable, the distances between the isobars along T vary linearly with $\ln p$ (see Figure VI-13). It may be seen that, if we take the isobars as approximately straight, which will be a good approximation for small regions, the tephigram may be considered as a skew emagram, with

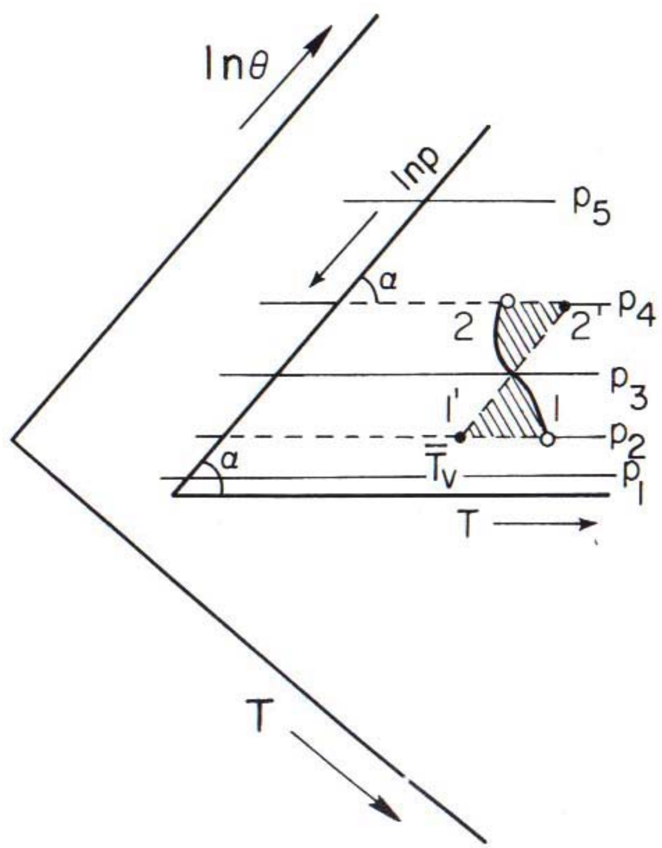

Fig. VI-13. The tephigram considered as an approximate skew-emagram.

an angle α between the axes of about 45°. The area determined on this diagram is

$$\sum\nolimits'_{\mathrm{te}} = \left[\int_1^2 T \, \mathrm{d} \ln p \right] \sin \alpha \tag{31}$$

which coincides with $\sum_{\mathrm{te}}$ except for the constant proportionality factor $\sin \alpha$. Within this approximation, we can therefore work with the tephigram as if it were an emagram, and apply the same methods as described before.

It should be noticed that the areas $\sum_{\mathrm{em}}$, $\sum_{\mathrm{te}}$ have been defined with the points 3, 4 on the vertical axis (origin of abscissae, $T=0$), which is usually outside the region represented in the diagram.

We can also remark that the area compensation in the tephigram will be exactly as good a procedure as in the emagram whenever the lines enclosing the areas have the same meaning, because the area equivalence is preserved. Thus, for instance, the slightly curved adiabat in the emagram and the straight adiabat in the tephigram will compensate areas equally well, provided in each case the adiabat passes through the point determined by $p = (p_1 p_2)^{1/2}$ and the same $\bar{T}_v$.

These methods can also be used for the computation of heights, and this is their main application, which will be considered in Chapter VIII, Section 11.

PROBLEMS

1. Construct a tephigram, drawing the following isopleths:
   $\theta = 250, 270, 290, 310, 330, 350$ K – Scale: 1 unit of $\log_{10} = 100$ cm.
   $T = 230, 250, 270, 290, 310$ K – Scale: 10 °C = 1.5 cm.
   $p = 1000, 900, 800, 700, 600, 500$ mb.
   $r_w = 1, 5, 10 \text{ g kg}^{-1}$.
   (For saturated adiabats, cf., Problem VII-8).
2. Starting from the emagram, and using the Jacobian, show that the tephigram is also an area-equivalent diagram, and that the proportionality factor between the area and the energy is c_p.
3. Show with the Jacobian that Stüve's diagram is not area-equivalent.
4. An air mass is defined by $T=20.0$ °C, $p=900$ mb, $U_w=70\%$. Find the following parameters on a tephigram: r, r_w, θ, T_d.
5. Assume that dry air undergoes a process which can be described by a straight line on a tephigram, going from (10 °C, 1000 mb) to (0 °C, 850 mb). Compute Δu, Δh and Δs, and apply the graphical methods of the mean temperature and of the mean adiabat to compute a and q.