These adiabats are labelled in terms of θ_w, (we may drop the subscript 'a' here without ambiguity), but the θ_w pseudo-adiabat only passes through (p, T) if the air is saturated. Let us define the saturation potential temperature, θ_s, as the value of θ_w for the pseudo-adiabat passing through (p, T), i.e., as the value of θ_w if the air were saturated. Mathematically,

$$\theta_s(p, T, r) = \theta_w(p, T, r_w), \quad \text{for} \quad r_w = r_w(p, T). \tag{88}$$

This is a rather artificial property, since it cannot be achieved by any simple physical process. It is, nevertheless, a rather useful concept, especially in air-mass analysis. Since most air masses have lapse rates close to the pseudo-adiabatic, a given frontal surface will be characterized by nearly constant values of θ_s. θ_s is conservative for saturated adiabatic processes, since it is then equal to θ_w, but is not conservative for dry adiabatic processes.

We therefore have conditional instability in any layer for which θ_s decreases with height, or for which

$$\sigma_s = -\frac{\delta \ln \theta_s}{\delta p} < 0. \tag{89}$$

We will have latent instability in any layer for which θ_s is less than θ_w at some lower level, as is evident from Figure IX-5 or any sounding on a thermodynamic diagram. Conditional instability without latent instability is essentially no instability at all; on the other hand, latent instability is impossible without conditional instability. Thus the presence of conditional instability indicates that one should examine the sounding more carefully for the possible existence of latent instability; if conditional instability is absent or marginal, a further stability analysis is not required.

One could carry out an analysis of stability changes with time in terms of the parameter, σ_s, so that $\partial\sigma_s/\partial t$ would correspond to decreasing stability or increasing conditional instability. However, since the only important class of conditional instability is latent instability, time changes of this latter phenomenon are more revealing and significant. Since the θ_s values and the θ_w values involved in latent instability refer to quite variable layers, in practice we would want to investigate $\partial\theta_s/\partial t$ in the middle troposphere and $\partial\theta_w/\partial t$ in the lower troposphere. Pressure levels of 500 and 850 mb, respectively, have been adopted for the Showalter Stability Index, useful for hail, tornado and thunderstorm forecasting; this index was defined at the end of Section 6. The Showalter Index is comparable to, but not identical with: $(\theta_s)_{500}-(\theta_w)_{850}$. It will always have the same sign as the above quantity; negative values are usually associated with severe thunderstorm situations.

If we have a layer of saturated air, there will be absolute instability if $\gamma > \gamma_s$ or if

$$\sigma_w = -\frac{\delta \ln \theta_w}{\delta p} < 0. \tag{90}$$

This instability would soon be released by turbulent overturning and mixing of the cloud air, a process that occurs in convective-type clouds, in which individual cells are often characterized by marked ascent or descent. It follows that saturated air is seldom observed with a lapse rate exceeding γ_s; but $\partial\sigma_w/\partial t < 0$ in saturated air would, of course, indicate the development of convective clouds, which are often observed to be imbedded in stratiform cloud decks.

Now let us consider the case of a layer of unsaturated air for which $\sigma_w < 0$, but $\sigma > 0$ and σ_s may be positive or negative. We may note in passing that if $\sigma_w > 0$ at all levels, θ_w can never exceed θ_s at a higher level, even though layers of conditional instability exist, since $\theta_s \geqslant \theta_w$. Thus, layers with $\sigma_w < 0$ and with $\sigma_s < 0$ (not necessarily the same layers) are necessary but not sufficient conditions for latent instability. If the layer with $\sigma_w < 0$ is now lifted to saturation, by a general ascent over a large area, instability can be released as soon as saturation is achieved. This we have defined in Section 10 as potential instability and it can only be realized by mass ascent. Latent instability, on the other hand, can be realized by parcel ascent, usually a convective or thermal process. For large scale energy release, however, general ascending motion is usually required; in subsiding air, convective activity is usually choked off and convective clouds seldom penetrate a subsidence inversion, for example.

In order to investigate $\partial\theta_s/\partial t$, $\partial\sigma_s/\partial t$, $\partial\theta_w/\partial t$ or $\partial\sigma_w/\partial t$, we require an analytic formulation for $\theta_w(p, T, r)$ and of $\theta_w(p, T, r_w)$, and thus must be able to integrate the pseudo-adiabatic equation, at least approximately, from p to $p_0 = 1000$ mb, and solve, directly or indirectly, for θ_w. Since we will be interested basically in $\partial\theta_w/\partial t$ and $\partial\theta_w/\partial p$, we may introduce approximations that would not be permitted if precise values of θ_w, alone, were to be the end product.

The simplest way to formulate θ_w or θ_s is to make use of the approximate equality between isobaric and adiabatic wet-bulb temperatures. We will assume that $T_{aw} = T_{iw} = T_w$ and $\theta_{aw} = \theta_{iw} = \theta_w$, where θ_w is the isobaric wet-bulb temperature of moist air at (p_0, θ, r) – for purposes of calculation – but is also considered to lie on the

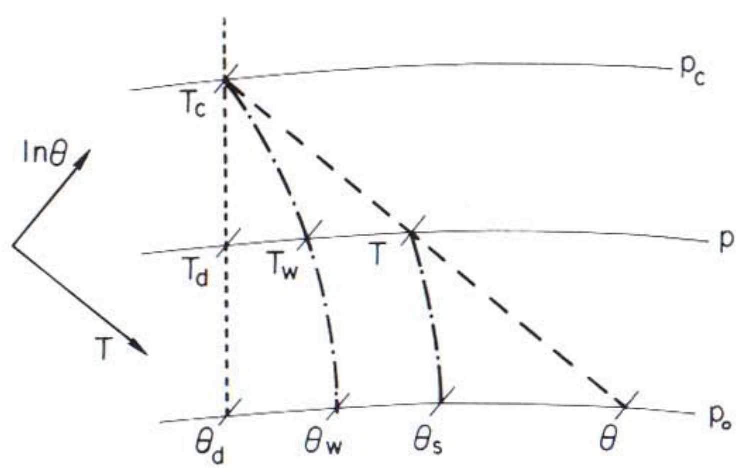

Fig. IX-18. Temperature parameters, on a tephigram.

same pseudo-adiabat as (p_c, T_c) and (p, T_{aw}), as illustrated on Figure IX-18, where dash-dot lines represent pseudo-adiabats and the dotted line a saturation mixing ratio line, corresponding to the actual mixing ratio of the air sample at (p, T). Along it, we can assert

$$r = r_w(p_c, T_c) = r_w(p, T_d) = r_w(p_0, \theta_d), \tag{91}$$

where we have defined a parameter θ_d, the dew-point potential temperature.

Consistent with the assumed equality of isobaric and adiabatic wet-bulb temperatures and potential temperatures, we may employ a simplified version of the Psychrometric Equation (Chapter VII, Equation (30)) and write, at p_0,

$$e_w(\theta_d) = e_w(\theta_w) - \frac{c_{p_d} p_0}{\varepsilon l_v}(\theta - \theta_w). \tag{92}$$

We may regard $\varepsilon l_v / c_{p_d} p_0$ as a constant and denote it by B, i.e.

$$B = \frac{\varepsilon l_v}{c_{p_d} p_0}. \tag{93}$$

Since

$$\frac{r}{r + \varepsilon} = \frac{e_w(\theta_d)}{p_0} = \frac{e_w(T_d)}{p} \tag{94}$$

we have, from Equations (92), (93) and (94),

$$\theta_w = \theta - B\left[e_w(\theta_w) - \frac{p_0}{p} e_w(T_d)\right]. \tag{95}$$

If the air at (p, T) is assumed to be saturated, as implied by the definition of θ_s, we have, by analogy,

$$\theta_s = \theta - B\left[e_w(\theta_s) - \frac{p_0}{p} e_w(T)\right]. \tag{96}$$

Separating out the properties of the original sample of moist air $-p$, T, T_d and θ, we can rewrite Equations (95) and (96) as

$$\theta_w + Be_w(\theta_w) = \theta + B\frac{p_0}{p} e_w(T_d) = \theta_e \tag{97}$$

$$\theta_s + Be_w(\theta_s) = \theta + B\frac{p_0}{p} e_w(T) = \theta_{se}. \tag{98}$$

In Equation (97), θ_e, a single-valued function of θ_w, is *not* the isobaric equivalent potential temperature, θ_{ie}, defined in (Chapter VII, Section 14), but is instead the isobaric equivalent temperature, at $p_0 = 1000$ mb, of air which has been taken dry-adiabatically to 1000 mb. In other words, θ_{ie} is not strictly conservative for dry adiabatic changes of state, whereas θ_{ae} is conservative for such a process (and so also is θ_e as defined by Equation (97)). These facts become apparent if we introduce Equation (93) into (Chapter VII, Equation (29)), giving

$$T_{ie} = T + B\frac{p_0}{p}e = T + B\frac{p_0}{p}e_w(T_d). \tag{99}$$

Introducing the definition of potential temperature, we have

$$\theta_{ie} = T_{ie}\left(\frac{p_0}{p}\right)^{\varkappa} = \theta + B\left(\frac{p_0}{p}\right)^{1+\varkappa} e_w(T_d) \tag{100}$$

whereas Equation (99) applied at p_0 gives, using Equation (94),

$$\theta_e = \theta + Be_w(\theta_d) = \theta + B\frac{p_0}{p}e_w(T_d),$$

which is identical to Equation (97). θ_e is much more useful than θ_{ie}, being conservative for a dry adiabatic process (for which θ and r are, in turn, conserved), and could be termed the isobaric-adiabatic equivalent potential temperature. There will be small changes in θ_e (but not in θ_{ae}) for saturated adiabatic ascent or descent, but these are less important from a practical standpoint in view of the uncertainties in the precise physical processes in operation.

In Equation (98), θ_{se}, a single-valued function of θ_s, may be called the saturation equivalent potential temperature. It depends only on the pressure and temperature of an air sample and can be computed, simply and directly in an analytic sense, from those parameters. Both θ_e and θ_{se} are ideally suited for a computer analysis of stability characteristics.

Now we may return to a consideration of stability criteria and stability changes, for the three most interesting classes of instability – conditional, potential and latent. Analogous to Equations (89) and (90), we may define

$$\sigma_e = -\frac{\delta \ln \theta_e}{\delta p} \quad \text{and} \quad \sigma_{se} = -\frac{\delta \ln \theta_{se}}{\delta p}. \tag{101}$$

We have conditional instability if

$$\gamma > \gamma_s \quad \text{or} \quad \sigma_s < 0 \quad \text{or} \quad \sigma_{se} < 0.$$

We have potential instability if

$$\sigma_w < 0 \quad \text{or} \quad \sigma_e < 0.$$

We have latent instability if

$$\theta_w > \theta_s(p') \quad \text{for some} \quad p' < p,$$

or if

$$\theta_e > \theta_{se}(p') \quad \text{for some} \quad p' < p.$$

We may now consider the physical processes that cause changes in the various types of instability being considered in this section. It will be convenient in formulating relations for such changes to neglect non-adiabatic effects, so that such effects will be discussed briefly first, in qualitative terms. Turbulent diffusion of heat and of water vapor in the vertical will always act to reduce potential stability if initially there is stability (and to reduce potential instability if initially there is instability), since the net result of vertical mixing processes is to reduce the vertical gradient of θ_e. On the other hand, turbulent interchange will favor the development of conditional instability. The other major non-adiabatic process is radiation, chiefly the long wave radiation of the earth and its atmosphere since solar radiation is absorbed primarily at the earth's surface, promoting instability of all types in the lower layers of the atmosphere. To appreciate the effects of long-wave (infrared) radiative exchange, we require information on the isobaric temperature changes which result from the vertical divergence of infrared radiative fluxes. When skies are clear, one can expect a cooling of the order of 1 to 2°C day^{-1} in the mid- and upper troposphere, somewhat less cooling in the lower troposphere and lower stratosphere and usually a minimum of cooling at the tropopause and a maximum (second maximum) at the earth's surface. When an overcast cloud layer is present, the atmospheric cooling is enhanced immediately above the cloud and greatly reduced below the cloud (and throughout the entire atmosphere below the cloud, unless the cloud is very high). The cloud itself will cool markedly if a low cloud and negligibly if a high cloud; the upper layers of the cloud always cool while the lower layers tend to warm, the net effect almost invariably being a cooling. Thus, with clear skies the effect of radiative cooling is to stabilize the atmosphere near the ground (offset by convective and conductive processes during the day), destabilize the middle and lower troposphere, stabilize the upper troposphere and destabilize the lower stratosphere. Radiative processes, like turbulent mixing processes, tend to reduce lapse-rate discontinuities and to smooth out bases and tops of inversion layers. When clouds are present, these effects are modified. Below an overcast layer, radiation acts to stabilize the atmosphere, and similarly above the cloud layer. Within the cloud layer, radiation acts to destabilize the stratification, and this effect may often be important in the development of nocturnal thunderstorms and of convective-type middle clouds. The cloud destabilization is intensified by the strong atmospheric cooling above the cloud and the warming below the cloud.

Returning now to adiabatic processes, the development of conditional instability is essentially associated with an increasing lapse rate (and a decrease in σ). Thus the processes discussed under the heading of stability changes for dry air (Section 11) apply here with equal force, and further elaboration is unnecessary. For the development of latent instability, one requires an increase in θ_w or θ_e at some level or a decrease in T or θ_s or θ_{se} at some higher level. At the higher level, ascent and advective cooling are the factors which assist in this process, whereas at the lower level horizontal advection of air of higher θ_w or θ_e is the important factor, vertical motions merely shifting in the vertical the level of maximum θ_w (or θ_e) along the trajectory. Expressed in another way, since θ_e is conserved for all adiabatic processes, local changes are due to horizontal and vertical advection.

The rate of change of potential stability (or instability) can be investigated by formulating $\partial\sigma_e/\partial t$, as we did $\partial\sigma/\partial t$ for dry air in Section 11, Equation (72) et seq., again employing an (x, y, p, t) coordinate set. Thus, from Equation (101),

$$\frac{\partial\sigma_e}{\partial t}=\frac{\partial}{\partial t}\left(-\frac{\partial\ln\theta_e}{\partial p}\right)=-\frac{\partial}{\partial p}\frac{\partial\ln\theta_e}{\partial t}, \tag{102}$$

and $\partial\sigma_e/\partial t$ will be positive for increasing potential stability or for decreasing potential instability. Since θ_e may be considered conserved following the motion of the air,

$$\frac{\mathrm{d}\ln\theta_e}{\mathrm{d}t}=0=\frac{\partial\ln\theta_e}{\partial t}+u\frac{\partial\ln\theta_e}{\partial x}+v\frac{\partial\ln\theta_e}{\partial y}+\frac{\mathrm{d}p}{\mathrm{d}t}\frac{\partial\ln\theta_e}{\partial p}. \tag{103}$$

Hence, from Equations (102) and (103) and using Equation (101)

$$\frac{\partial\sigma_e}{\partial t}=\frac{\partial}{\partial p}\left(u\frac{\partial\ln\theta_e}{\partial x}+v\frac{\partial\ln\theta_e}{\partial y}\right)-\frac{\partial}{\partial p}\left(\sigma_e\frac{\mathrm{d}p}{\mathrm{d}t}\right). \tag{104}$$

Carrying out the differentiation implied by Equation (104), we have, again using Equation (101),

$$\frac{\partial\sigma_e}{\partial t}=\left(\frac{\partial u}{\partial p}\frac{\partial\ln\theta_e}{\partial x}+\frac{\partial v}{\partial p}\frac{\partial\ln\theta_e}{\partial y}\right)-\left(u\frac{\partial\sigma_e}{\partial x}+v\frac{\partial\sigma_e}{\partial y}\right)-\sigma_e\frac{\partial}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t}-\frac{\mathrm{d}p}{\mathrm{d}t}\frac{\partial\sigma_e}{\partial p}. \tag{105}$$

In order to evaluate the first term, above, we must differentiate Equation (97), at constant pressure. Thus,

$$\left(\frac{\partial\ln\theta_e}{\partial x}\right)_p=\frac{1}{\theta_e}\left\{\left(\frac{\partial\theta}{\partial x}\right)_p+B\frac{p_0}{p}\frac{\mathrm{d}e_w(T_d)}{\mathrm{d}T_d}\left(\frac{\partial T_d}{\partial x}\right)_p\right\}. \tag{106}$$

Introducing the equation for potential temperature and the Clausius-Clapeyron equation,

$$\frac{\partial\ln\theta_e}{\partial x}=\frac{1}{\theta_e}\left(\frac{p_0}{p}\right)^{\varkappa}\frac{\partial T}{\partial x}+\frac{Bp_0}{\theta_e p}\frac{\varepsilon e_w(T_d)l_v}{R_d T_d^2}\frac{\partial T_d}{\partial x}. \tag{107}$$

Writing a similar equation for $\partial\ln\theta_e/\partial y$, and using Equation (93) we have

$$\frac{\partial u}{\partial p}\frac{\partial\ln\theta_e}{\partial x}+\frac{\partial v}{\partial p}\frac{\partial\ln\theta_e}{\partial y}=\frac{1}{\theta_e}\left(\frac{p_0}{p}\right)^{\varkappa}\left(\frac{\partial u}{\partial p}\frac{\partial T}{\partial x}+\frac{\partial v}{\partial p}\frac{\partial T}{\partial y}\right)+$$

$$+\frac{\varepsilon^2 l_v^2 e_w(T_d)}{c_{p_d}R_d\theta_e pT_d^2}\left(\frac{\partial u}{\partial p}\frac{\partial T_d}{\partial x}+\frac{\partial v}{\partial p}\frac{\partial T_d}{\partial y}\right). \tag{108}$$

The first term was examined for the dry air stability tendency analysis in Equation

(76), by introducing the isobaric shear of the geostrophic wind

$$\frac{\partial T}{\partial x} = -\frac{fp}{R_d}\frac{\partial v_g}{\partial p} \quad \text{and} \quad \frac{\partial T}{\partial y} = \frac{fp}{R_d}\frac{\partial u_g}{\partial p}. \tag{109}$$

Thus, the first term in Equation (108) vanishes for geostrophic or gradient (i.e., non-accelerated) winds, and can be neglected in general. The remaining term in Equation (108) vanishes for saturated air, since then $T_d = T$, but for unsaturated air it may be significantly positive or negative (we will return to this term shortly).

The second term in Equation (105) represents the isobaric advection of potential stability (or instability) and the fourth term the vertical advection. The remaining term can be written, using Equation (83), as

$$-\sigma_e \frac{\partial}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t} = \sigma_e\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \tag{110}$$

and is thus the vertical shrinking or stretching term (or, the horizontal divergence or convergence term, respectively).

As in the case of dry-air stability, it is instructive to consider the degree of conservation of potential stability, following a trajectory, for the case of quasi-gradient flow and quasi-adiabatic conditions. We commence with the formal relation for $\mathrm{d}\theta_e/\mathrm{d}t$, i.e.,

$$\frac{\mathrm{d}\theta_e}{\mathrm{d}t} = \frac{\partial\theta_e}{\partial t} + u\frac{\partial\theta_e}{\partial x} + v\frac{\partial\theta_e}{\partial y} + \frac{dp}{\mathrm{d}t}\frac{\partial\theta_e}{\partial p}. \tag{111}$$

Introducing Equation (111) into (105), substituting Equation (108) and (110), and making the geostrophic assumption via Equation (109), into wind shear terms but not convergence terms, we obtain

$$\frac{\mathrm{d}\theta_e}{\mathrm{d}t} = \sigma_e\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\varepsilon^2 l_v^2 e_w(T_d)}{f c_{p_d}\theta_e p^2 T_d^2}\left(\frac{\partial T}{\partial y}\frac{\partial T_d}{\partial x} - \frac{\partial T}{\partial x}\frac{\partial T_d}{\partial y}\right). \tag{112}$$

These two terms express the only mechanisms (apart from non-adiabatic effects or strongly accelerated flow) for a changing potential stability in a moving element of air. The first term predicts that divergence (by which we mean horizontal, or more strictly, isobaric divergence) will increase either potential stability or instability, whereas convergence decreases both potential stability and instability. However, such a process can never convert potential stability to potential instability, or vice versa. Thus the final term is very important, since in an air mass with potential stability everywhere it can create potential instability, under the appropriate conditions of temperature and dew point gradients (on an isobaric surface). This term can be considered to represent the effect of advection of dew point by the thermal wind (an appellation applied to the vector shear of the geostrophic wind, the orthogonal components of this shear being related to the thermal field by the set (109)).

To clarify this concept, let us recall that the (x, y) axes represent a right-handed cartesian set of axes. Conventionally they are chosen, in meteorology, with x increasing to the east and y to the north. Since the orientation is really arbitrary, let us select the x-axis as parallel to isotherms, with temperature decreasing in the direction of y increasing (see Figure IX-19, in which solid lines represent isotherms and dashed

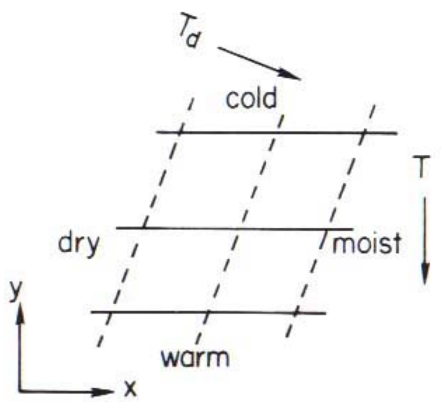

Fig. IX-19. Synoptic situation conducive to creation of potential instability.

lines represent lines of constant dew point). From Equation (109), we see that the thermal wind 'blows' in the direction of the positive x-axis (the wind component in this direction increasing with height, with the orthogonal component constant with height). From Equation (112), we see that decreasing potential stability (or increasing potential instability) requires that $\partial T_d/\partial x$ be positive, i.e., that the dew point increase in the direction of the thermal wind. In other words, the thermal wind must advect drier air, essentially with a true advection proceeding more rapidly above the given level than below. This synoptic situation is not uncommon east of the Rocky Mountains in the United States, especially in summer, with maritime tropical air from the Gulf of Mexico to the east and continental tropical air from the southwestern (arid) states to the west. When a strong thermal-wind advection of drier air takes place in a situation with general ascent to release the instability, hail and tornadoes often occur.

9.13. Radiative Processes and Their Thermodynamic Consequences

Let us consider first radiative processes near the ground, with clear skies. During most of the daylight period, the earth's surface gains energy by radiation, the solar radiation absorbed exceeding the net loss of infrared radiation. This radiative energy gain is dissipated in three ways – by heating of the ground, by heating of the air and by evaporation from the surface. Frequently, it is possible to forecast the heat input into the air, and from this energy (Q_a) one may estimate the probable maximum temperature. Let us assume that the sounding at a time of minimum temperature, $T_1(p)$, is known or can be estimated, and that the sounding at time of maximum temperature, $T_2(p)$, is characterized by a dry-adiabatic lapse rate up to the level where diurnal changes are small, as shown in Figure IX-20.

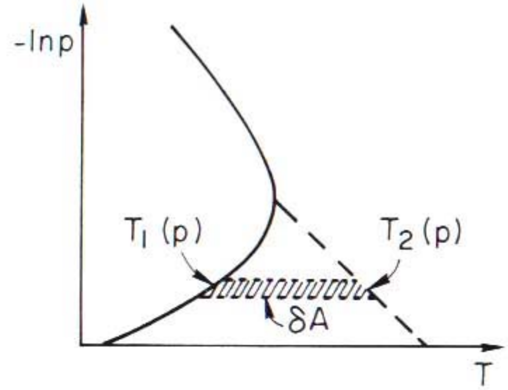

Fig. IX-20. Effect of solar radiation on temperature stratification.

Let us define an element of area, δA, on the emagram in Figure IX-20, as

$$\delta A = -(T_2 - T_1)\,\mathrm{d}\ln p\,. \tag{113}$$

From the First Principle of Thermodynamics, the heat input into a cylindrical box of unit cross-section for this infinitesimally thin layer is

$$\delta Q_a = -c_p(T_2 - T_1)\frac{\mathrm{d}p}{g}. \tag{114}$$

Introducing Equation (113) into (114) and integrating over the entire atmosphere (from surface pressure, p_s, to zero pressure)

$$Q_a = \frac{c_p}{g}\int_{p=p_s}^{p=0} p\,\delta A\,. \tag{115}$$

From the Theorem of the Mean, this can be rewritten as

$$Q_a = \frac{c_p}{g}\,\bar{p}A\,. \tag{116}$$

A similar relation applies to any thermodynamic diagram. It will be recalled that an area on such a diagram is equal to the work performed for a cyclic process undergone by unit mass about that area. In this case we are not dealing with unit mass, but with the entire lower atmosphere, so that the area, A, is not simply equal nor proportional to the energy input Q_a.

After sunset, solar radiation need no longer be considered and the net loss of energy at the surface due to infrared radiation must be balanced by fluxes down from the air and up from the ground, the downward heat flux including that of latent heat associated with condensation at the surface, visible as dew if the surface itself cannot diffuse water into the ground at a sufficiently rapid rate. Since the lapse rate in the lowest layers in the early morning hours depends on many complex factors, it is not fruitful to attempt to forecast the appropriate area on a thermodynamic diagram comparable to A, above, nor to obtain the minimum temperature by such an approach. A direct solution of the heat conduction equations is possible with appropriate simplifications, giving a forecast of the minimum temperature at the ground (the most critical parameter for frost-damage assessment). If this temperature is expected to fall below the dew point of the air, fog formation (radiation fog) is very likely. The effect of the resultant condensation on air temperature and on liquid water content of the fog can be estimated by the procedures of (Chapter VII, Section 5), or by their analytical analogues (i.e., finding the isobaric wet-bulb temperature of supersaturated air rather than the adiabatic wet-bulb temperature). From (Chapter VII, Equation (29)),

$$T_w - T = \frac{l_v}{c_{p_d}}(r - r_w), \tag{117}$$

where T represents the expected minimum temperature (neglecting the possibility of condensation), T_w the actual minimum temperature (with fog containing a liquid water mixing ratio of $(r-r_w)$, r the saturation mixing ratio at the dew point temperature (we assume $T_d > T$) and r_w the saturation mixing ratio at the final temperature T_w. We may therefore state that

$$r - r_w = \left(\frac{\partial r_w}{\partial T}\right)_p (T_d - T_w). \tag{118}$$

Introducing $r_w \simeq \varepsilon l_w/p$ and the Clausius-Clapeyron equation, Equation (118) becomes

$$r - r_w = \frac{\varepsilon r l_v}{R_d T_d^2}(T_d - T_w). \tag{119}$$

From Equations (117) and (119),

$$(T_w - T_d) + (T_d - T) = \frac{\varepsilon l_v^2 r}{R_d c_{p_d} T_d^2}(T_d - T_w).$$

Thus,

$$(T_d - T_w)\left(1 + \frac{\varepsilon r l_v^2}{c_{p_d} R_d T_d^2}\right) = (T_d - T). \tag{120}$$

From Equation (120), $(T_d - T_w)$ can be found, and then $(r - r_w)$, from Equation (119). An estimate of the horizontal visibility may be made from the following table:

$(r - r_w)$ in g kg^{-1}:	0.015	0.025	0.065	0.09	0.15	0.25	0.35	0.65	1.8
Visibility in m:	900	600	300	240	180	120	90	60	30

Once a radiation fog has formed, it will tend to thicken as long as radiational cooling of the fog plus ground can continue, since a dense fog acts as a black body as far as infrared radiation is concerned. Thus the major further cooling takes place at the fog top, and the maximum fog density may even be at some distance above ground level. After sunrise, a dense fog may continue to cool since the solar radiation is largely reflected and little absorbed by the fog. The ground itself will commence to warm slowly (as a result of the radiation transmitted through the fog), and the increased turbulence will initially mix the fog near the ground. At times this process may actually produce a further decrease in surface visibility shortly after sunrise. Eventually, however, the rising surface temperature and the net gain of radiational energy by the fog itself will lead to fog dissipation. For a thick fog, the fog will tend to dissipate first at the surface, leaving a low layer of stratus cloud above. This cloud becomes progressively thinner and less dense as the lower layers continue to warm, and disappears completely when the level of cloud top (earlier, the level of fog top) has the same potential temperature as the entire layer below.

Overcast cloud layers above the layer of diurnal temperature changes will normally exhibit a net cooling as a result of radiative processes, although there may be a slight gain of energy during the middle of the day. Since the cloud top acts as a black body to infrared radiation (but definitely not to solar radiation), the atmosphere immediately above the cloud behaves, for most if not all of the 24 h, like the air above the ground at night, except that the analogue to fog formation is of course a cloud-thickening process.

Finally, let us consider the consequences of radiative cooling of broken or scattered clouds, particularly when relatively thin. For a given cloud height, the radiative cooling rate is inversely proportional to cloud depth, to a first approximation. For a thin cloud, this cooling rate may far exceed that of the air between clouds, so that the cloud soon becomes denser than its environment and thus subsides, pseudo-adiabatically, until it reaches equilibrium with the environment at a lower level. This process normally continues until the liquid water content of the cloud is entirely depleted and the cloud has dissipated; this process is responsible for most cases of nocturnal cloud dissipation, e.g., for scattered or broken stratocumulus, etc., in the early evening. The net effect of the cooling plus subsidence is that the cloud descends along the environment curve, but a number of different cases may arise, depending on the ambient lapse rates and the cloud liquid water content.

Various possible cases will be illustrated by schematic tephigram curves. CS

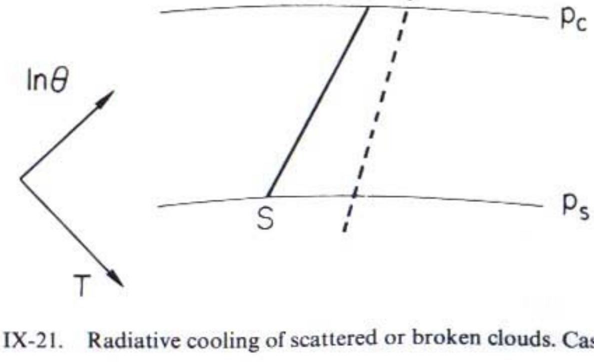

Fig. IX-21. Radiative cooling of scattered or broken clouds. Case I.

sets a virtual limit very close to the dew point, an important fact in forescasting minimum temperatures.

For an isobaric process, the heat absorbed is given by the increase in enthalpy (Chapter IV, Equation (106)):

$$\delta q = \mathrm{d}h \cong c_\mathrm{p}\,\mathrm{d}T + l_\mathrm{v}\,\mathrm{d}r\,. \tag{15}$$

If we write $r \cong \varepsilon e/p$, as p is a constant, $\mathrm{d}r \cong (\varepsilon/p)\,\mathrm{d}e$. In this case e corresponds to saturation, and we may apply the Clausius-Clapeyron Equation (Chapter IV, Equation (48)) and write $e = e_\mathrm{w}$; we obtain:

$$\mathrm{d}r \cong \frac{\varepsilon}{p}\,\mathrm{d}e = \frac{\varepsilon l_\mathrm{v} e_\mathrm{w}}{pR_\mathrm{v}T^2}\,\mathrm{d}T \tag{16}$$

and

$$\delta q = \left(c_\mathrm{p} + \frac{\varepsilon l_\mathrm{v}^2 e_\mathrm{w}}{pR_\mathrm{v}T^2}\right)\mathrm{d}T, \tag{17}$$

or else

$$\delta q = \left(\frac{c_\mathrm{p} R_\mathrm{v} T^2}{l_\mathrm{v} e_\mathrm{w}} + \frac{\varepsilon l_\mathrm{v}}{p}\right)\mathrm{d}e_\mathrm{w}\,. \tag{18}$$

The relation between $\mathrm{d}T$ and $\mathrm{d}e_\mathrm{w}$ is indicated in Figure VII-4, on a vapor pressure diagram.

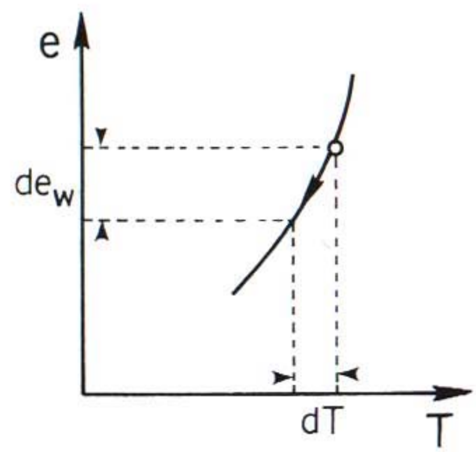

Fig. VII-4. Relation between the changes in temperature and in vapor pressure during condensation.

If we compute the heat loss $-\delta q$ from other data (e.g.: radiation loss), Equation (17) allows an estimation of the corresponding decrease in temperature $-\mathrm{d}T$. Similarly, the decrease in vapor pressure $-\mathrm{d}e_\mathrm{w}$ may be computed from Equation (18). From the gas law, the mass of water vapor per unit volume is given by $e_\mathrm{w}/R_\mathrm{v}T$, and its variation with temperature is $\mathrm{d}e_\mathrm{w}/R_\mathrm{v}T - (e_\mathrm{w}/R_\mathrm{v}T^2)\,\mathrm{d}T$. Introducing the Clausius-Clapeyron

Case IIIa, $\gamma_w > \bar{\gamma} > \gamma_r$ and $r_w(T_s) < r_t$ (Figure IX-23). In this case (for which a representative pseudo-adiabat is labeled as θ_w), the cloud does not dissipate completely as it sinks. Instead, the cloud density decreases and the cloud reaches the ground as thin fog patches. The increased turbulent mixing in this case, compared to cases I and II, could well lead to cloud dissipation by mixing with the environment air (especially if the latter is relatively dry).

Case IIIb, $\gamma_w > \bar{\gamma} > \gamma_r$ and $r_w(T_s) > r_t$ (Figure IX-24). In this case the cloud descends only to P, where $r_w = r_t$. The cloud has dissipated at this point and the descending volume is in equilibrium with the environment.

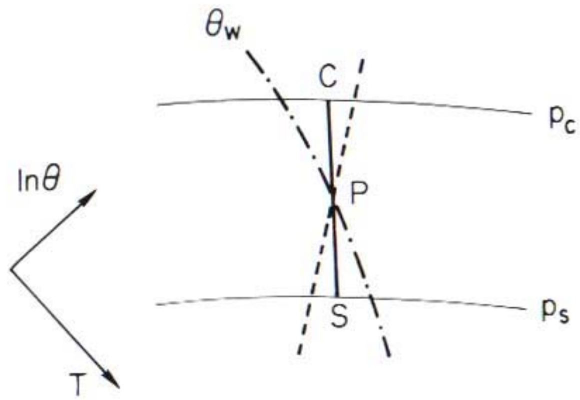

Fig. IX-24. Radiative cooling of scattered or broken clouds. Case IIIb.

If $\gamma_w > \bar{\gamma}$, the atmospheric stratification is stable for ascent or descent of saturated air, with or without a condensed phase. If $\gamma_d > \bar{\gamma} > \gamma_w$, the atmospheric stratification is stable for ascent or descent of unsaturated air, is unstable for ascent of saturated air, with or without condensed phase, and is unstable for descent of saturated air containing liquid water (or ice). Clouds produced by convection (cumulus) or by vertical mixing (stratocumulus) will have nearly dry-adiabatic lapse rates below the cloud, at least during the active formation stage, and in addition it is possible to have $\bar{\gamma} > \gamma_w$ below clouds produced by orographic ascent or by the general ascending motion associated with the stratified clouds in cyclonic weather systems. In general, the liquid water content at the base of all such clouds is small, so that descending motions triggered by instability soon lead to dissolution of such cloud elements (within the drier air below cloud base), but this does not contribute to dissipation of the cloud as a whole if active formation processes are simultaneously in operation. Radiative cooling of broken or scattered clouds under these conditions (which, we note, do not apply to overcast layers) will accelerate the dissipation of the entire cloud system, and this process is undoubtedly a potent one in the evening hours when the effects of convection and vertical mixing are greatly reduced, i.e., for cumulus and stratocumulus.

Case IVa, $\gamma_d > \bar{\gamma}\gamma_w$ and $\theta(P) > \theta(S)$ (Figure IX-25). With slight cooling, the cloud becomes unstable for descent and subsides rapidly to P, where it dissipates. The ex-cloud air at P is denser than the environment, and sinks dry-adiabatically to R, where it is in equilibrium with the environment.

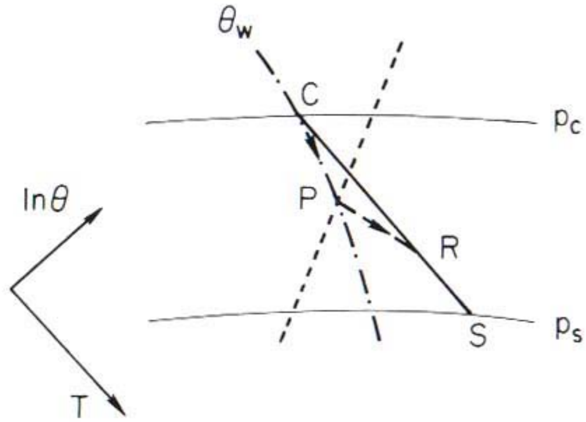

Fig. IX-25. Radiative cooling of scattered or broken clouds. Case IVa.

Case IVb, $\gamma_d > \bar{\gamma} > \gamma_w$ and $\theta(P) < \theta(S)$ (Figure IX-26). As before, the cloud cools slightly and descends rapidly along a pseudo-adiabat to P, where it dissipates completely. The ex-cloud air at P now sinks dry-adiabatically right down to the ground (at R), producing local cool down-drafts.

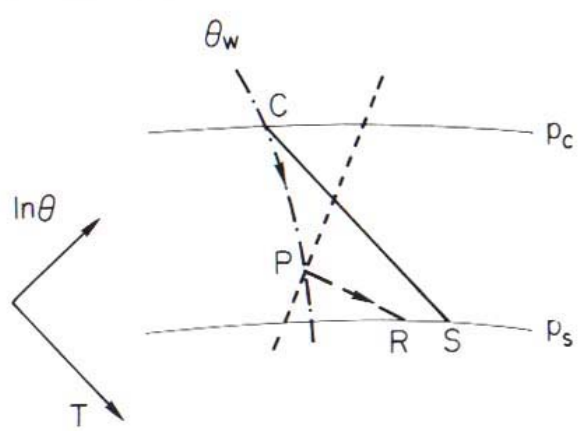

Fig. IX-26. Radiative cooling of scattered or broken clouds. Case IVb.

We may now compute the total amount of heat (Q_r) that must be lost by radiative processes in order that the cloud may be dissolved or reach the ground, whichever occurs first. During the first stage of the stepwise process, isobaric cooling accompanied by latent heat release, the first principle of thermodynamics (cf. Chapter IV, Equation (104)) gives, ignoring the heat content of the vapor and of the condensed phase,

$$\delta Q_r = c_{p_d}\,\mathrm{d}T_1 + l_v\,\mathrm{d}r_w = c_{p_d}\,\mathrm{d}T_1 + l_v\left(\frac{\partial r_w}{\partial T}\right)_p \mathrm{d}T_1\,. \tag{121}$$

During the second stage of the stepwise process, the reversible saturated adiabatic descent (the process analyzed in Chapter VII, Section 8), we may approximate the corresponding relation by neglecting the variation of l_v/T, giving

$$c_{p_d}\,dT_2 - R_d T\, d\ln p_2 + l_v\, dr_w = 0. \tag{122}$$

Expansion of dr_w in Equation (122) gives

$$c_{p_d}\,dT_2 + l_v\left(\frac{\partial r_w}{\partial T}\right)_p dT_2 - R_d T\, d\ln p_2 + l_v\left(\frac{\partial r_w}{\partial p}\right)_T dp_2 = 0. \tag{123}$$

Let us now add Equations (121) and (123), and introduce $\delta T = dT_1 + dT_2$ as the temperature change of the environment air for an infinitesimal pressure change, δp. Since $\delta p = dp_2$, it follows that

$$\delta Q_r = c_{p_d}\,\delta T + l_v\left\{\left(\frac{\partial r_w}{\partial T}\right)_p \delta T + \left(\frac{\partial r_w}{\partial p}\right)_T \delta p\right\} - R_d T\,\delta\ln p. \tag{124}$$

Introducing the hydrostatic Equation (Chapter VIII, Equation (21)), and denoting by δr_w the change of saturation mixing ratio along the environment curve (for an increment $\delta\varphi$, of geopotential), we have

$$\delta Q_r = c_{p_d}\,\delta T + l_v\,\delta r_w + \delta\varphi. \tag{125}$$

This equation can now be integrated over the layer from C to S (or C to P, in case IIIb), treating l_v as a constant and computing the change in geopotential by the method of the mean adiabat (Chapter VIII, Section 11), i.e., from

$$\delta\varphi = -\,c_{p_d}\,\delta T_0. \tag{126}$$

The resulting value of Q_r is the heat which must be dissipated, per unit mass of the cloud. This should be converted to the heat loss per unit area of the entire cloud, to be consistent with the convention for radiative fluxes.

9.14. Maximum Rate of Precipitation

Let us consider now a unit mass of saturated air rising in the atmosphere. During the ascent, vapor will condense into water (or ice); by imagining that all the water precipitates as rain (or snow), we may compute an upper limit for the possible rate of precipitation. We shall proceed now to make this computation.

We start from Chapter VII, Equation (84) for the pseudoadiabats. In its last term, l_v can be treated as a constant without much error. We then develop

$$d\left(\frac{r_w}{T}\right) = \frac{dr_w}{T} - \frac{r_w}{T^2}\,dT,$$

multiply the whole equation by T and make the substitution $R_d T\, d\ln p = -d\phi$. We then obtain:

$$\left[c_{p_d} + \left(c_w - \frac{l_v}{T}\right) r_w\right] dT + d\phi + l_v\, dr_w = 0.$$

The second term within the square bracket usually amounts to some units per cent of c_{p_d} (it reaches 10% at $r_w \cong 0.02$) and will be neglected in this approximate treatment. The equation can then be rearranged into:

$$-\frac{dr_w}{d\phi} = \frac{1 - (\gamma_w/\gamma_d)}{l_v} \tag{127}$$

where Equation (12) has been taken into account, as well as the fact that $-dT/d\phi = \gamma_w$ in our case.

Equation (127) gives the condensed water per unit geopotential of ascent. We shall now assume that we have a saturated layer $\delta\phi$ thick, rising at a velocity U. The mass of air contained in this layer per unit area is $\varrho\,\delta z = \delta\phi/gv$.

We consider that the derivative with minus sign

$$-\frac{dr_w}{dt} = -\frac{dr_w}{d\phi}\frac{d\phi}{dt} \tag{128}$$

expresses the mass of condensed water per unit mass of air and unit time. We have also:

$$\frac{d\phi}{dt} = g\frac{dz}{dt} = gU\,. \tag{129}$$

Introducing this expression and Equation (127) in (128), and multiplying by the mass of air, we shall have the mass of water condensed (and eventually precipitated) per unit time and unit area:

$$P = \frac{1 - (\gamma_w/\gamma_d)}{l_v v}\,U\,\delta\phi \tag{130}$$

P is here given, if MKS units are used, in kg m^{-2} s^{-1}. The mass of water precipitated per unit area may be expressed by the depth that it fills, in mm. Taking into account that the density is 10^3 kg m^{-3}, 1 kg m^{-2} of precipitated water is equivalent to 1 mm. If we further refer the precipitation to 1 h rather than to 1 s, we must multiply the rate of precipitation by 3600 in order to have it expressed in mm h^{-1}, as is common practice in meteorology:

$$P = 3600\,\frac{1 - (\gamma_w/\gamma_d)}{l_v v}\,U\,\delta\phi \quad (\text{mm h}^{-1}) \tag{131}$$

where all quantities are in MKS units.

The coefficient of $U\,\delta\phi$ in Equation (131) depends only on T and p, and so will P for a given value of U and $\delta\phi$. Therefore they will be defined for each point of a diagram. We can then choose, for instance, $U = 1$ m s^{-1} and $\delta\phi = 100$ gpm $= 981$ J kg^{-1}, and draw isopleths of constant P. If we are considering an ascending saturated layer at (T, p), we can read P on the diagram, and this will give us the maximum precipitation rate for each m s^{-1} of vertical velocity and each 100 gpm of thickness.

A similar computation may be performed for snow precipitation.

Figure IX-27 shows the shape of these isopleths on a tephigram. They are labeled with the value of P and 'S' or 'R' for 'snow' and 'rain', respectively.

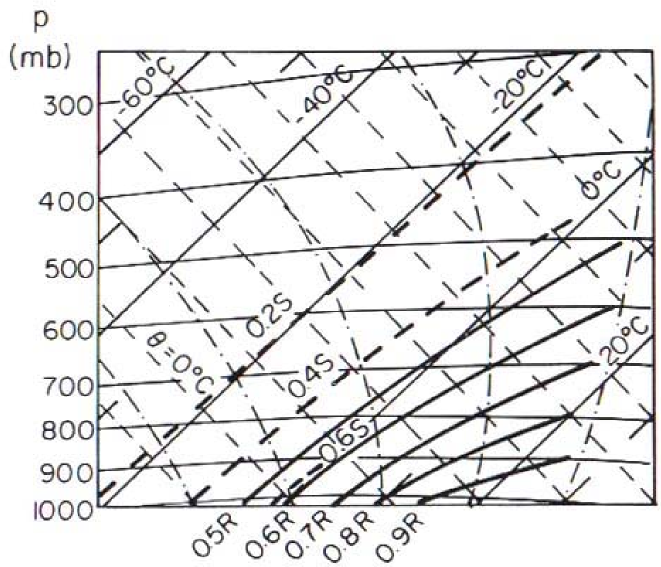

Fig. IX-27. Isopleths of maximum precipitation rate, on a tephigram. R: rain; S: snow.

Let us consider now a technique for the evaluation of the maximum rate of precipitation in terms of the distribution in the vertical of the vertical motion in isobaric coordinates, $\mathrm{d}p/\mathrm{d}t$. This turns out to be particularly straightforward on a tephigram, but could be adapted readily to any other thermodynamic diagram.

We may state, completely analogous to Equation (130), above,

$$P = \int \frac{\mathrm{d}r_w}{\mathrm{d}t} \frac{\mathrm{d}p}{g}, \tag{132}$$

where the integration is taken over the entire cloud layer (or layers) to give the precipitation rate in mass of water per unit time per unit area.

Introducing Chapter IV, Equation (114) into Equation (122), we have

$$-\frac{\mathrm{d}r_w}{\mathrm{d}t} = \frac{T}{l_v} \frac{\mathrm{d}s_d}{\mathrm{d}t} \tag{133}$$

in terms of the dry-air entropy, s_d – a basic coordinate of the tephigram. The time derivative is that associated with the pseudo-adiabatic process, which implies that

$$\frac{\mathrm{d}s_d}{\mathrm{d}t} = \left(\frac{\partial s_d}{\partial p}\right)_{\theta_w} \frac{\mathrm{d}p}{\mathrm{d}t}. \tag{134}$$

Substituting Equations (133) and (134) into (132), we obtain the required integral, which may also be expressed as a summation over finite layers, e.g.,

$$P = -\int \frac{T}{gl_v}\frac{\mathrm{d}p}{\mathrm{d}t}\left(\frac{\partial s_d}{\partial p}\right)_{\theta_w}\mathrm{d}p = \sum\left(\frac{T}{gl_v}\right)\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)(\Delta s_d)_{\theta_w}, \tag{135}$$

where mean values are employed of T/l_v and of $\mathrm{d}p/\mathrm{d}t$ for each layer, and where $(\Delta s_d)_{\theta_w}$ represents the dry-air entropy decrease along a mean pseudo-adiabat downward through the layer in question (we note this can be read off directly, on a tephigram, and is by definition a negative quantity).

In practice, isobaric vertical motions can be deduced from numerical weather prediction calculations, in either a diagnostic or prognostic mode (in the latter case, a prognostic sounding would also have to be employed, plus an indirect estimation of the depth of saturated layers). In theory, at least, we could employ observed rates of precipitation in order to verify the accuracy of computation of vertical motions. This is not a satisfactory procedure, however, since precipitation amounts are seldom representative, and a few rain gauges do not constitute an adequate sample of a large area for quantitative purposes. If instability phenomena are present, and their existence is not always obvious from synoptic weather data, the above equation will seriously underpredict precipitation, and in this latter instance horizontal variability will be large and random, increasing the difficulty of verification. Moreover, it will seldom be obvious from vertical soundings what was the precise vertical extent of the cloud layers (even if verification is being carried out with diagnostic vertical motions). Finally, the assumptions made (constant cloud properties, no evaporation below the cloud) could seldom be justified in individual cases. Thus, the equation for P is useful chiefly for semi-quantitative prediction for a large area.

9.15. Internal and Potential Energy in the Atmosphere

The Earth as a whole is in a state of radiative equilibrium, in which the solar radiation absorbed is compensated by emission to space, essentially as infrared radiation corresponding to surface and atmospheric temperatures.

Between absorption and emission, however, there is a complicated pattern of energy transformations. The heating by the Sun causes an increase in the specific internal energy of the atmospheric gas. As the gas expands due to the increase in temperature, it lifts the mass center of any vertical column that may be considered; this implies an increase in potential energy. There is also a partial transformation of the radiant energy received into vaporization enthalpy. The internal and potential energies may be partially transformed into kinetic energy of motion of large air masses, into turbulence energy, into mechanical work over the Earth's surface and finally into heat. The last conversion implies again an increase in internal and potential energy, thus closing a cycle. Other transformations between these different forms of energy are also possible. Differential latitudinal heating and horizontal transport are implied in this complex picture.

We have the following basic types of energy in the atmosphere: (1) potential (gravitational), (2) internal (thermal), and (3) kinetic. If we consider the approximate formulas Chapter IV, Equations (111) and (105):

$$u = c_v T + l_v q + \text{const.}$$

$$h = c_p T + l_v q + \text{const.}$$

we can see that the internal energy and the enthalpy will each include a term proportional, at every point, to the temperature, and another due to the vaporization heat of its water vapor content. It is customary to refer to the former as 'sensible heat', and to the latter as 'latent heat'. The kinetic energy can appear in large scale motions, in vertical convection or in a whole spectrum of eddies.

Here we shall only consider the expressions of potential and internal energy, their inter-relation and their possible transformation into kinetic energy of vertical motions. Moreover, we shall only consider in the internal energy and in the enthalpy the term proportional to the temperature, as we shall not study the more complex case when there are water phase transitions. We shall call U, P and K the internal, potential and kinetic energy, respectively, and H, as usual, the enthalpy.

Let us consider a vertical atmospheric column of unit cross section, in hydrostatic equilibrium, extending from the surface ($z=0$) to the height h. Each infinitesimal layer $\mathrm{d}z$ has a mass $\varrho\,\mathrm{d}z$ and the internal energy $c_v T\varrho\,\mathrm{d}z$*. We shall have for the column (introducing the hydrostatic equilibrium equation Chapter VIII, Equation (19)):

$$U = c_v \int_0^h T\varrho\,\mathrm{d}z = \frac{c_v}{gR}\int_0^{\phi_h} p\,\mathrm{d}\phi = \frac{c_v}{g}\int_{p_h}^{p_0} T\,\mathrm{d}p \tag{136}$$

where the small variations of c_v and R with humidity and of g with altitude are neglected.

The same integration can be performed for the enthalpy. It is easily seen that $H = \eta U$.

For the potential energy:

$$P = \int_0^h \phi\varrho\,\mathrm{d}z = g\int_0^h \varrho z\,\mathrm{d}z = \int_{p_h}^{p_0} z\,\mathrm{d}p\,. \tag{137}$$

Integrating by parts and introducing the gas law:

$$P = -\,p_h h + R\int_0^h \varrho T\,\mathrm{d}z = -\,p_h h + \frac{R}{c_v}\,U = -\,p_h h + (\eta - 1)\,U\,. \tag{138}$$

* We are taking for convenience the arbitrary additive constant as zero, which amounts to considering as reference state one obtained by extrapolation of the formula $u = c_v T$ to $T = 0$ K.

If the column extends to the top of the atmosphere, $p_h = 0$ and Equation (138) becomes

$$P = (\eta - 1)U \tag{139}$$

a proportionality which must always be obeyed, provided there is hydrostatic equilibrium. As $\eta = 7/5$, we have the relation

$$P : U = 2 : 5. \tag{140}$$

If the column receives heat from external sources, it must become distributed between the potential and the internal energy in the same proportion; i.e., $2/7 = 29\%$ must go to increase P and $5/7 = 71\%$ to increase U.

If we add both energies for such a column, we obtain

$$P + U = \eta U = H \tag{141}$$

where the last equation results by considering that ηU is given by expressions similar to Equation (136), but with c_p instead of c_v, in the coefficient. Thus, for a column in hydrostatic equilibrium extending in height to negligible pressures, the total enthalpy is equal to the sum of the internal and potential energies, and to η times the internal energy. Obviously, any energy received from external sources produces an equivalent increase in the enthalpy.

Now let us consider an infinite column divided, for the purpose of the discussion, into a lower part extending from ground to $z = h$ corresponding to a constant given P_h and with energies P, U, and an upper part, from $z = h$ upwards and with energies P', U' (see Figure IX-28). Let P_t, U_t be the values for the total column. The lower part must obey Equation (138). For the upper part, performing the same integration, but this time between the limits $z = h$ and $z = \infty$ (viz. from $p = p_h$ to $p = 0$), we arrive at

$$P' = p_h h + (\eta - 1)U'. \tag{142}$$

The sum of Equations (138) and (142) gives

$$P + P' = (\eta - 1)(U + U') \tag{143}$$

i.e.,

$$P_t = (\eta - 1)U_t \tag{144}$$

which is again the Equation (139), as applied to the present case. We notice now that for a process which does not alter the atmospheric structure above P_h (such as absorption of heat by the lower part), U' remains constant, while P' changes because of the change in h:

$$\Delta P' = p_h \Delta h \tag{145}$$

where p_h, which represents the weight of the whole column of unit cross section above the initial level h, must remain constant. That is, the change in P' is equal to the change in potential energy that would be experienced by a solid weight p_h (per unit cross section) resting on top of the lower part of the column, as this top is raised by Δh.

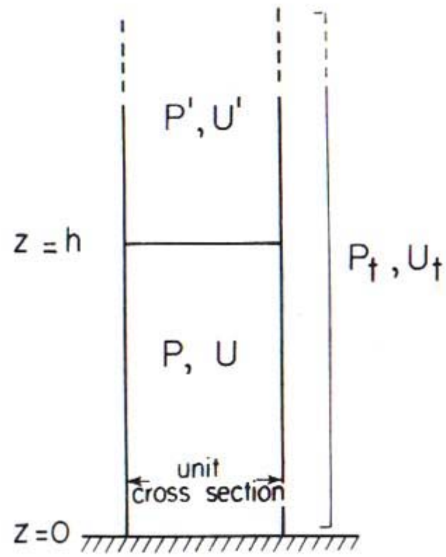

Fig. IX-28. Internal and potential energy of an atmospheric column.

From Equations (138), (144), and (145),

$$\Delta P_t = \Delta(P + p_h h) = (\eta - 1)\Delta U . \tag{146}$$

Summarizing, any energy input will go entirely to an increase $\Delta H_t = \Delta(P_t + U)$, from which the fraction 5/7 goes to ΔU and 2/7 to ΔP_t; from this latter portion, $p_h\,\Delta h$ gives the increase $\Delta P'$ and the rest goes to ΔP.

If we now consider an adiabatic process in which there are no motions with a component normal to the limiting surfaces of the system, and we neglect the work of external friction forces, we must have

$$\Delta(K + P_t + U) = 0 \tag{147}$$

$$\Delta K = -\Delta(P_t + U) = -\eta\Delta U = -\Delta H. \tag{148}$$

That is, the kinetic energy that can be produced is given by the decrease in the sum $H_t = (P_t + U)$ when passing from the initial to the final stratification. This is the maximum value attainable in principle, as no real process will be strictly adiabatic; there will be, in a larger or lesser degree, simultaneous reconversion of kinetic into internal and potential energy through frictional dissipation.

9.16. Internal and Potential Energy of a Layer with Constant Lapse Rate

Let us consider an atmospheric column in hydrostatic equilibrium with a thermal gradient $\gamma = \text{const.}$, extending from $z = 0$ to $z = h$. Using the equations, Chapter VIII, Equation (37) and Chapter VIII, Equation (40) to integrate the second expression (136) of

U, we obtain:

$$U = \frac{c_v}{gR} p_0 \int_0^{\phi_h} \left(1 - \frac{\gamma}{T_0}\phi\right)^{1/R\gamma} d\phi$$

$$= -\frac{c_v p_0 T_0}{gR\gamma} \left. \frac{\left(1 - \frac{\gamma}{T_0}\phi\right)^{(1/R\gamma)+1}}{\frac{1}{R\gamma} + 1} \right|_0^{\phi_h} \tag{149}$$

$$= \frac{c_v}{g(1 + R\gamma)} (p_0 T_0 - p_h T_h),$$

where real temperatures and gradients are used, instead of the virtual ones.

For the particular case of an adiabatic layer, $\gamma = 1/c_p$ and $R\gamma = \varkappa$. We then have

$$U = \frac{c_v}{g(1 + \varkappa)} (p_0 T_0 - p_h T_h)$$

$$= \frac{c_p}{g(2\eta - 1)} (p_0 T_0 - p_h T_h). \tag{150}$$

And, writing the temperature in terms of the pressure and the potential temperature

$$T_0 = \theta \left(\frac{p_0}{p_{00}}\right)^{\varkappa}; \qquad T_h = \theta \left(\frac{p_h}{p_{00}}\right)^{\varkappa}$$

we obtain

$$U = \frac{c_v \theta}{g(1 + \varkappa) p_{00}^{\varkappa}} (p_0^{\varkappa+1} - p_h^{\varkappa+1}), \tag{151}$$

where $p_{00} = 1000$ mb. Therefore, for an adiabatic layer and given p_0 and p_h, the internal energy is proportional to its potential temperature, θ.

9.17. Margules' Calculations on Overturning of Air Masses

Margules used the previous formulas to calculate the change in potential and internal energy and to estimate the possible production of kinetic energy, in the following idealized case.

We shall assume two air layers of unit cross section, superimposed, both adiabatic, with the potential temperature of the upper layer θ_2 smaller than that of the lower layer θ_1, so that the system is unstable. p_0, p_m and p_h are the pressures at the bottom of the column, at the interface between the two layers and at the top, respectively (see Figure IX-29).

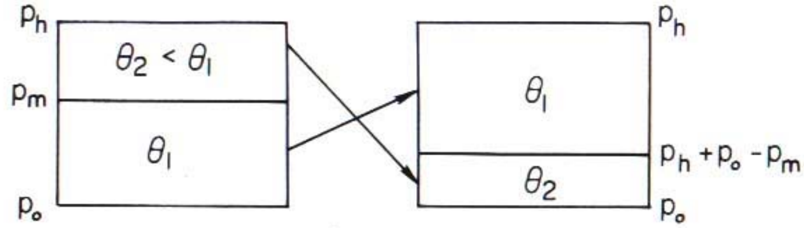

Fig. IX-29. Overturning of two adiabatic layers, according to Margules.

We assume now that an overturning takes place, by which the two layers exchange positions, but without any mixing taking place between the air masses. It is easy to see that the pressures at the bottom and at the top will be the same, but at the interface between the two layers it will now be $(p_h + p_0 - p_m)$, because this surface is holding the weight of the atmosphere above the whole column, given by p_h, and that of the layer θ_1, given (from the initial state) by $p_0 - p_m$.

We now apply Equation (151) to calculate the initial (U_i) and final (U_f) internal energies of the system

$$U_i = \frac{c_v}{g(1+\varkappa)p_{00}^{\varkappa}} \left[\theta_1 (p_0^{\varkappa+1} - p_m^{\varkappa+1}) + \theta_2 (p_m^{\varkappa+1} - p_h^{\varkappa+1})\right] \tag{152}$$

$$U_f = \frac{c_v}{g(1+\varkappa)p_{00}^{\varkappa}} \{\theta_1 [(p_h + p_0 - p_m)^{\varkappa+1} - p_h^{\varkappa+1}] + \\ + \theta_2 [p_0^{\varkappa+1} - (p_h + p_0 - p_m)^{\varkappa+1}]\}. \tag{153}$$

The variation of U in the overturning will therefore be:

$$\Delta U = \frac{c_v}{g(1+\varkappa)p_{00}^{\varkappa}} (\theta_1 - \theta_2) \times \\ \times [(p_h + p_0 - p_m)^{\varkappa+1} + p_m^{\varkappa+1} - p_0^{\varkappa+1} - p_h^{\varkappa+1}], \tag{154}$$

which can also be written:

$$\Delta U = -\frac{c_v}{g(1+\varkappa)} \left(\frac{p_0}{p_{00}}\right)^{\varkappa} (\theta_1 - \theta_2) \left[p_0 + p_h \left(\frac{p_h}{p_0}\right)^{\varkappa} - p_m \left(\frac{p_m}{p_0}\right)^{\varkappa} - \\ - (p_0 + p_h - p_m) \left(\frac{p_0 + p_h - p_m}{p_0}\right)^{\varkappa}\right]. \tag{155}$$

In the case of layers of moderate thickness, so that p_0 and p_h do not differ greatly, the following approximations can be made in Equation (155):

$$\left(\frac{p}{p_0}\right)^{\varkappa} = \left(1 + \frac{p - p_0}{p_0}\right)^{\varkappa} = 1 + \varkappa \frac{p - p_0}{p_0} + \ldots \cong 1 + \varkappa \frac{p - p_0}{p_0}.$$

Introducing this simplification, Equation (155) becomes:

$$\Delta U \cong -\frac{2a}{g}\left(\frac{p_{00}}{p_0}\right)^{1-\varkappa}(\theta_1 - \theta_2)\frac{(p_0 - p_m)(p_m - p_h)}{p_{00}} \tag{156}$$

with

$$a = \frac{c_v \varkappa}{1+\varkappa} = \frac{R}{\eta(1+\varkappa)} = \frac{R}{2\eta - 1}.$$

Equation (156) shows that the change in internal energy will be proportional to the difference of potential temperatures and to the product of the pressure thicknesses of the two layers. The same proportionality will hold, according to Equations (146) and (148), for ΔP_t and for ΔK, with the appropriate change of coefficients. In particular:

$$\Delta K \cong \frac{2a\eta}{g}\left(\frac{p_{00}}{p_0}\right)^{1-\varkappa}(\theta_1 - \theta_2)\frac{(p_0 - p_m)(p_m - p_h)}{p_{00}}. \tag{157}$$

This is the maximum value of kinetic energy that can be produced, in principle, during the overturning; if the total system is isolated, this kinetic energy will finally dissipate again into internal and potential energy (with changes in θ_1, θ_2 or both). We can use the value obtained in Equation (157) to compute the velocity W related to it by

$$\Delta K = \tfrac{1}{2} M W^2 \tag{158}$$

where $M = (p_0 - p_h)/g$ is the total mass (per unit cross section). As this is such an idealized model, we are only interested in the resulting order of magnitude. For layers 100 mb thick, with potential temperatures differing by 10 °C, we obtain $W = 21\ \mathrm{m\ s^{-1}}$; with 200 mb, $W = 30\ \mathrm{m\ s^{-1}}$. This is the order of magnitude of the strongest updraughts in storms, but we must remember that in real storms there is condensation, the updraughts are localized over certain areas, and the whole process is much more complex and implies a great deal of turbulent mixing.

Computation shows that the values of ΔK are only a small fraction of U. For instance, with $\theta_1 = 300$ K, $\theta_2 = 290$ K and 100 mb thick layers starting at 1000 mb, ΔK is about 920 times (and ΔU about 1290 times) smaller than $U = 4.2 \times 10^8\ \mathrm{J\ m^{-2}}$.

Margules also calculated the variation of potential and internal energy for the case of adjacent layers, as shown in Figure IX-30, and obtained similar results.

Fig. IX-30. Overturning of adjacent adiabatic layers.

9.18. Transformations of a Layer with Constant Lapse Rate

Let us consider a layer extending from $z=0$ to $z=h$, with a constant lapse rate which in general will be different from γ_d. Its internal energy will be given by Equation (149).

We shall only consider qualitatively the processes that may occur in this layer.

(a) If vertical mixing takes place, the resulting potential temperature will be uniform and given by the weighted average θ through all the layer (cf. Chapter VII, Section 12). The initial lapse rate is γ and the final one is the adiabatic lapse rate γ_d. It may be shown that according to whether $\gamma \gtrless \gamma_d$, the variation of internal energy will be $\Delta U \lesseqgtr 0$. Thus, if the layer had a superadiabatic lapse rate, the mixing may occur spontaneously, with a decrease in potential and internal energy, and the layer will acquire an adiabatic lapse rate. In this case, there will be a surplus of energy which, assuming that the process occurs without exchange with the environment, will be first transformed into kinetic energy and then dissipated into heat by turbulence. The final potential temperature of the layer will be such as to maintain the initial value of U and P. If $\gamma = \gamma_d$, the layer is in neutral equilibrium with respect to vertical exchanges.

If $\gamma < \gamma_d$, the layer is stable; in order to produce the vertical mixing, which occurs with an increase in potential and internal energy, the layer must absorb energy from external sources.

(b) Littwin considered another ideal process in a layer with uniform superadiabatic lapse rate γ_i: the total orderly overturning of the layer, in such a way that the stratification is inverted, with the highest layers passing to the lowest positions and vice-versa. The final gradient γ_f is subadiabatic. During the overturning, each infinitesimal layer follows a dry adiabat to its new location. It is assumed that no mixing occurs. The process occurs with decrease in potential and internal energy. Figure IX-31 illustrates this case; it may be noticed that a discontinuity has been assumed at p_h, to indicate that the upper part (stable), does not participate in the process.

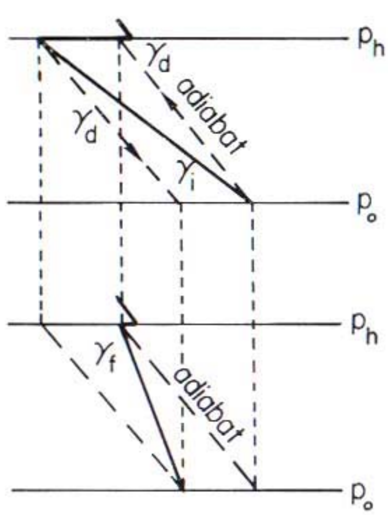

Fig. IX-31. Overturning of a layer with a superadiabatic constant lapse rate.

As Margules did, Littwin computed the kinetic energy corresponding to the decrease in $(P_t + U)$, and from it, the mean velocities attainable during the overturning, with results of the same order as of the Margules calculation, i.e. similar to our results based on Equation (158).

Obviously Littwin's process, although more elaborate than Margules', still is an unrealistic idealized model. Particularly, the condition of absence of mixing will not hold in the atmosphere. Real processes will be intermediate between (a) and (b), and in particular for local convection they may be closer to case (a); actually we have seen that convection caused by ground heating during strong insolation leads to thorough mixing and an adiabatic lapse rate in the lowest layers.

9.19. The Available Potential Energy

As we have already remarked, the pioneering calculations of Margules were aimed at understanding the source of the energy of storms and based on crude models, rather inadequate when considered as a local representation of strong convective processes. But the same concepts have been carried over to large-scale motions, mainly by Lorenz, where they find fruitful application and can be related to the problems of the general circulation and of the energy conversions in the atmosphere. We shall not develop his theory, which would be beyond the scope of this book, but shall mention briefly the basic ideas and indicate how the concepts of the previous sections can be generalized.

Again we shall restrict consideration to processes in which condensation plays no role (or its role is minor and can be ignored) and shall assume that conditions of vertical hydrostatic equilibrium are essentially valid everywhere. Equations (136) to (141) are therefore applicable over each particular location. If we now integrate the expressions of U and P for columns extending to the top of the atmosphere (i.e., to $p = 0$) over an extended surface or over the whole surface of the earth, we shall have the total values of U and P for that extended region or for the whole atmosphere, respectively. It is customary, in the context of the present problem to call the sum $(P + U)$ the *total potential energy*. It must be stressed, however, that as Equation (141) is valid at every location, it is also valid for the integrated value over the whole surface under consideration; therefore the total potential energy is identically the *total enthalpy* $H = P + U$.

We shall define *adiabatic motion* (or more generally *adiabatic process*) as a motion (or process) in which entropy, and therefore potential temperature, is conserved for every parcel of air.

Atmospheric motion is not in general adiabatic; friction over the ground surface or in the air and mixing are nonadiabatic processes. However, friction is the only nonadiabatic process which directly alters the kinetic energy, destroying it and generating internal energy. The remaining nonadiabatic processes alter only the internal energy directly. So the only source of kinetic energy K in the atmosphere is $H = P + U$.

The advantage of considering adiabatic flows resides in that, in spite of the previous remarks, they represent a good approximation to large-scale motions in the atmosphere.

In an adiabatic process, an air parcel moves on an isentropic surface, i.e., a constant θ

surface. If the parcel is to be accelerated, there must be a pressure gradient on the surface; without it, the kinetic energy of the parcel will remain constant. Therefore, in a situation where constant p surfaces and constant θ surfaces coincide, there can be no conversion of H into K, no matter how large H is. The question that becomes important then is what is the portion of H that can be converted into K. This is indeed a very difficult question.

An easier question, however, can be asked. For a given state of the atmosphere, what is the minimum value of the total enthalpy (or total potential energy) that can be attained by an adiabatic redistribution of mass in the atmosphere? We call this minimum value H_{min}, and we define now the *available potential energy* A as the following difference:

$$A = H - H_{min}. \tag{159}$$

It will be noticed that we are considering now for the whole atmosphere (or a large portion of it) a process similar to the local adiabatic overturnings of Margules and Littwin from an initial, unstable stratification to a final stable one, where U and H have minimum values.

The advantage of defining A in this manner lies in that it separates the part relevant to the production of air motions from the much larger bulk of total enthalpy, mostly unavailable for conversion into kinetic energy. The convenience of doing so can be appreciated in the following two examples: (a) in an atmosphere with horizontal, hydrostatically stable stratification, however large H may be, $A = 0$; (b) on the other hand, removing heat differentially from this atmosphere will decrease H, but will create $A > 0$, causing instability (i.e., will decrease H_{min} more than it does H).

It is clear that the available potential energy has the following properties:

(1) $(A + K)$ is conserved under adiabatic flow.

(2) A is completely determined by the distribution of mass.

(3) $A = 0$ if the stratification is such that constant p and constant θ coincide everywhere.

(4) $A > 0$ if constant p and constant θ surfaces do not coincide.

According to property (1), A can be considered as the only source of kinetic energy:

$$\Delta(A + K) = 0. \tag{160}$$

Equation (160) is the generalized equivalent of our previous Equation (147). It is important to recognize that, for a given situation, not all of A may be converted into K in the real atmosphere, because the redistribution of mass required to achieve H_{min} may not satisfy the equation of motion. The available potential energy A merely gives an upper bound for the enthalpy to kinetic energy conversion. In fact, it can be estimated that typically only about 1/10 of the total enthalpy of the atmosphere is transformed into kinetic energy. As the available potential energy is found to be of the order of 1/200 of the total enthalpy, only about 1/2000 of the latter transforms into K.

Equation (147) indicates that A is the only sink for K in adiabatic flow, but, as mentioned before, in real flows friction will dissipate part of the kinetic energy, producing an increase in $H_{\min}$ but not in A.

Now let us derive an explicit expression for A.

Since θ increases monotonically with height (according to the assumed stable vertical hydrostatic equilibrium), we shall use θ instead of p or z as the vertical coordinate. $p(x, y, \theta)$ may be considered as the weight of air with potential temperature exceeding θ, at x, y. This is true even if $\theta < \theta_0$, where θ_0 is the surface potential temperature (i.e., extending formally the range of the variable θ_0 to underground locations), provided that we define $p(x, y, \theta) = p_0(x, y)$, where p_0 is the surface pressure, for $\theta < \theta_0$.

The average of p over an isentropic surface of area S is

$$\bar{p}(\theta) = \frac{1}{S}\int_S p(x, y, \theta)\,\mathrm{d}S \tag{161}$$

$\bar{p}$ is conserved under adiabatic redistribution of mass, because $S\bar{p}$ gives the total weight of air with potential temperature exceeding θ, which is conserved.

The total enthalpy for the area S is

$$H = \frac{c_p}{g}\int_S\int_0^{p_0} T\,\mathrm{d}p\,\mathrm{d}S = \frac{c_p}{g p_{00}^{\varkappa}}\int_S\int_0^{p_0}\theta p^{\varkappa}\,\mathrm{d}p\,\mathrm{d}S$$

$$= \frac{c_p}{g(1+\varkappa)p_{00}^{\varkappa}}\int_S\left[\int_0^{p_0}\theta\,\mathrm{d}p^{1+\varkappa}\right]\mathrm{d}S \tag{162}$$

where $p_{00} = 1000$ mb and c_p and g are considered as constants. Let us discuss the integral between the bracket; solving by parts:

$$\int_0^{p_0}\theta\,\mathrm{d}p^{1+\varkappa} = \theta p^{1+\varkappa}\Big|_0^{p_0} - \int_{\theta_0}^{\infty} p^{1+\varkappa}\,\mathrm{d}\theta$$

$$= \theta_0 p_0^{1+\varkappa} - \int_{\theta_0}^{\infty} p^{1+\varkappa}\,\mathrm{d}\theta \tag{163}$$

where the lower limit value of the first integral at the right-hand side has been set equal to 0 because p decreases with height more rapidly (essentially as an exponential variation) than θ increases. Now, according to the condition mentioned above for the extension of the range of θ, p remains constant and equal to p_0 for $\theta < \theta_0$, so that

$$\theta_0 p_0^{1+\varkappa} = \int_0^{\theta} p^{1+\varkappa}\,\mathrm{d}\theta \tag{164}$$

and (163) becomes

$$\int_0^{p_0} \theta \, \mathrm{d}p^{1+\varkappa} = \int_0^{\infty} p^{1+\varkappa} \, \mathrm{d}\theta. \tag{165}$$

Introducing (165) into (162):

$$H = \frac{c_p}{g(1+\varkappa)p_{00}^{\varkappa}} \int_S \int_0^{\infty} p^{1+\varkappa} \, \mathrm{d}\theta \, \mathrm{d}S. \tag{166}$$

Now $H_{\min}$ can be achieved by rearranging mass in such a way that p is constant on the isentropic surfaces, and this constant p should be equal to the earlier defined $\bar{p}$, because $\bar{p}$ is conserved under adiabatic processes. Therefore

$$H_{\min} = \frac{c_p}{g(1+\varkappa)p_{00}^{\varkappa}} \int_S \int_0^{\infty} \bar{p}^{1+\varkappa} \, \mathrm{d}\theta \, \mathrm{d}S \tag{167}$$

and

$$A = \frac{c_p}{g(1+\varkappa)p_{00}^{\varkappa}} \int_S \int_0^{\infty} (p^{1+\varkappa} - \bar{p}^{1+\varkappa}) \, \mathrm{d}\theta \, \mathrm{d}S. \tag{168}$$

We can write

$$p = \bar{p} + p' \tag{169}$$

where usually $p' \ll p$; this allows us to make the approximation

$$p^{1+\varkappa} = \bar{p}^{1+\varkappa}\left(1 + \frac{p'}{\bar{p}}\right)^{1+\varkappa} \cong \bar{p}^{1+\varkappa} \times \times \left[1 + (1+\varkappa)\frac{p'}{\bar{p}} + \frac{(1+\varkappa)\varkappa}{2}\left(\frac{p'}{\bar{p}}\right)^2\right] \tag{170}$$

which, introduced into (168), gives

$$A = \frac{c_p}{g(1+\varkappa)p_{00}^{\varkappa}} \left[(1+\varkappa) \int_S \int_0^{\infty} \bar{p}^{1+\varkappa} \frac{p'}{p} \, \mathrm{d}\theta \, \mathrm{d}S + \frac{(1+\varkappa)\varkappa}{2} \int_S \int_0^{\infty} \bar{p}^{1+\varkappa} \left(\frac{p'}{\bar{p}}\right)^2 \mathrm{d}\theta \, \mathrm{d}S\right]. \tag{171}$$

The first integral within the bracket vanishes, as can be readily seen by replacing p' from (169), integrating first over the surface ($\bar{p}$ being a constant for this integration) and considering (161). Equation (171) reduces therefore to

$$A = \frac{c_p \varkappa}{2g p_{00}^{\varkappa}} \int_S \int_0^{\infty} \bar{p}^{\,1+\varkappa} \left(\frac{p'}{\bar{p}}\right)^2 \mathrm{d}\theta \, \mathrm{d}S. \tag{172}$$

If we want to express A in terms of temperature, we write

$$T = \bar{T} + T' \tag{173}$$

where $\bar{T}$, the average temperature over an isentropic surface, is defined in a similar way to $\bar{p}$ (Equation (161)) and $T' \ll T$. On each isentropic surface, we have the relations:

$$T = \theta \left(\frac{p}{p_{00}}\right)^{\varkappa} = \theta \left(\frac{\bar{p}}{p_{00}}\right)^{\varkappa} \left(1 + \frac{p'}{\bar{p}}\right)^{\varkappa} \simeq \bar{T}\left(1 + \varkappa \frac{p'}{p}\right). \tag{174}$$

Therefore

$$\frac{p'}{\bar{p}} = \frac{1}{\varkappa} \frac{T'}{\bar{T}} \tag{175}$$

which, introduced into (172), gives finally

$$A = \frac{c_p}{2g \varkappa p_{00}^{\varkappa}} \int_S \int_0^{\infty} \bar{p}^{\,1+\varkappa} \left(\frac{T'}{\bar{T}}\right)^2 \mathrm{d}\theta \, \mathrm{d}S. \tag{176}$$

Let us consider an example based on a simplified model. We assume a rectangular surface of length L and width W. We take the variable x along the length and y along the width. All parameters will be assumed independent of y.

Figure IX-32 shows a plot of constant p surfaces in the x, θ plane. With respect to the

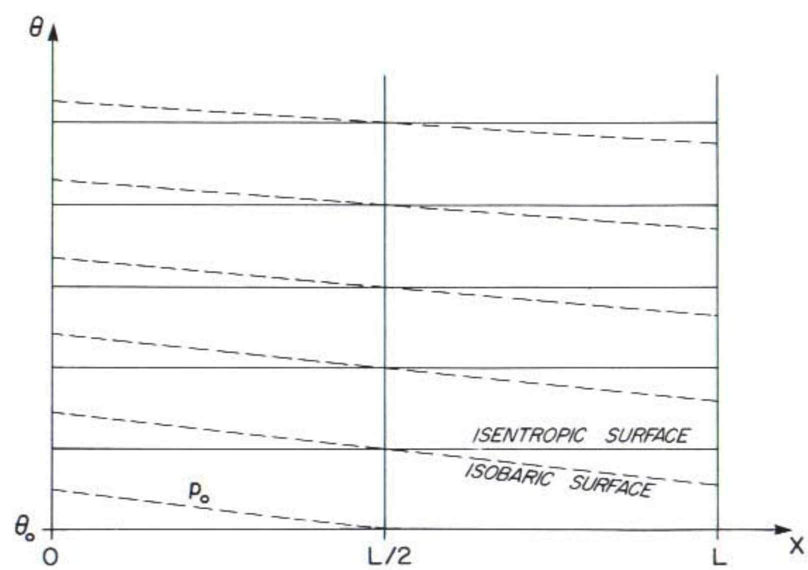

Fig. IX-32.

horizontal isentropic surfaces, the isobaric surfaces are assumed to be inclined with a constant slope, according to the linear variation

$$p(x,\theta) = p\left(\frac{L}{2},\theta\right)\left[1 + s\left(1 - \frac{2x}{L}\right)\right] \tag{177}$$

where s is a constant determining the slope. We further assume that the lapse rate is constant throughout the atmosphere at $x = L/2$, with a value γ. The air is assumed dry for convenience (or humidity corrections are neglected). The solution of the example is left to Problem 18. Here we shall only give the results, assuming the following parameters. The surface pressure at the midpoint is $p_0(L/2, \theta_0) = 1000$ mb, the surface potential temperature is $\theta_0 = 300$, K, $\gamma = 6.5$ K gpkm^{-1} and $s = 0.15$ (which means that the pressures on the isentropic surface θ_0 vary up to $\pm$ 150 mb). The calculation shows that the available potential energy and the total enthalpy, referred to unit area, are:

$$A = 3.56 \times 10^6 \text{ J m}^{-2}, \qquad H = 2.58 \times 10^9 \text{ J m}^{-2}$$

and

$$A/H = 1/725.$$

PROBLEMS

1. Prove that the work of expansion of an air parcel ascending adiabatically and quasi-statically may be expressed by the following formula:

$$\delta a = -\frac{gT}{\eta T_e}\,\mathrm{d}z$$

where T_e is the temperature of the environment, $\eta = c_p/c_v$, and a is referred to unit mass.

2. From the following sounding

p(mb)	T(°C)	U_w%	p(mb)	T(°C)	U_w%
920	24.0	68	568	− 1.2	42
900	22.5	70	545	− 2.0	49
850	20.0	83	500	− 8.0	77
800	15.8	83	400	− 19.5	71
765	13.0	92	300	− 33.0	
735	12.8	55	250	− 41.5	
700	10.0	54	200	− 54.0	
645	5.8	55	150	− 65.5	
600	3.0	32			

(a) Compute the positive and negative instability areas for a surface parcel, on a tephigram.

(b) Compute the same for a parcel at 600 mb.

(c) Determine the thickness (base and top pressures) of the layer with latent instability.

(d) Determine the lifting condensation level (LCL) and the free convection level (FCL) for a surface parcel.

(e) Compute the instability index, defined as the temperature of a parcel from 850 mb when taken to the 500 mb level minus the temperature of the atmosphere at the same level.

(f) Assuming that after the time of the sounding and due to insolation, the lower layer absorbs heat from the ground and cumuli start developing, what will be the level of the cloud base?

3. Given the following temperature sounding

p(mb)	T(°C)
950	22.5
900	18.0
850	15.0
800	16.0
750	12.0
700	7.0
650	4.0
600	− 1.5
500	− 10.0
400	− 20.0

and knowing that the dew-point at the surface (950 mb) is 15.7 °C, plot the sounding on a tephigram and

(a) Determine, using the tephigram, the mixing ratio r, the relative humidity U_w, the potential temperature θ, the wet-bulb temperature T_{aw}, the potential wet-bulb temperature θ_{aw} and the potential equivalent temperature θ_{ae} for a surface parcel. Mark the relevant points on the diagram. Compute the isobaric wet-bulb temperature T_{iw} and equivalent temperature T_{ie}.

(b) Determine the lifting condensation level and the free convection level for a surface parcel. What can you say of the conditional instability of surface parcels?

4. Plot the following points on an aerological diagram:

p(mb)	T(°C)	r(g kg^{-1})
1000	17.0	10.0
900	–	9.0
850	8.0	6.0
800	–	3.8
700	− 4.5	–
500	− 20.0	–
300	− 35.0	–

Join them by straight lines for each variable, and assume that the two representations obtained correspond to an atmospheric sounding. Find the potential temperature θ, the wet-bulb potential temperature θ_w, the lifting condensation level (LCL), the saturation temperature T_s and the free convection level (FCL) for a ground (1000 mb) parcel, and mark on the diagram the negative and positive areas of instability for vertical parcel displacement. Find the convective condensation level (CCL). Indicate the layer with latent instability.

5. The vertical distribution of temperature and humidity (mixing ratio r) of the atmosphere over a certain location is given initially by the following data:

p(mb)	T(°C)	r(g kg^{-1})
1000	20.0	11.5
850	12.0	9.0
700	2.0	5.0
600	− 5.5	2.5
500	− 14.5	1.5

Radiative heating of the ground results in the development of convection, attainment of the convective condensation level (CCL) and cumuli formation. Using a tephigram, find:

(a) The lifting condensation level (LCL) for a ground parcel before the heating, and the CCL.

(b) The dew point, the pseudo-wet bulb temperature and the potential pseudo-wet bulb temperature for ground parcels before and after the heating.

(c) The approximate vertical velocity acquired by an air parcel in a cumulus at 600 mb, as predicted by the parcel theory. Assume that the initial velocity at the CCL is negligible and use the area equivalence given for the diagram.

(d) Indicate the layer with latent instability at the initial time (before the heating).

6. Given the following data:

p(mb):	1000	900	800	700	600	500	400
T(°C):	25	18	10	2	−4.5	−12	−20

and knowing that the relative humidity at the ground (1000 mb) is 60% and that the mixing ratio has an average value of 4 g kg^{-1} between 600 and 800 mb;

(a) determine, on a tephigram, the values of the following parameters for ground parcels: adiabatic potential wet bulb temperature θ_{aw}, dew point temperature T_d, lifting condensation level (LCL) and free convection level (FCL).

(b) What type of conditional instability do these parcels have?

(c) On a separate tephigram, compute the thickness of the layer between 800 and 600 mb, in a single step, both by the method of mean temperature and of the mean adiabat. (Use the tephigram as if it were a skew emagram.) Express the results in gpm and in m^2 s^{-2}.

7. The potential pseudo-wet bulb temperature decreases with height through an atmospheric layer ($\delta\theta_{aw}/\delta z < 0$). What comments can you make on its vertical stability, if the layer is saturated? And if it is not saturated?
8. An unsaturated air mass is ascending through the isobar p. Due to radiant heat exchange, the virtual potential temperature increases by 2.8 K gpkm^{-1}.

   Determine the state of stability of the atmosphere at that level if its (geometric) virtual temperature lapse rate is 7.3 K gpkm, and
   (a) if $p = 800$ mb;
   (b) if $p = 500$ mb.
9. A vertically ascending air particle is receiving some heat. The temperature of the particle is increasing by 0.0134 K m^{-1}. Determine
   (a) the polytropic exponent n and
   (b) the heat flux.
10. A parcel of unsaturated air is receiving heat from the surroundings through heat conduction a rate of $\delta q/\mathrm{d}z$ during its ascent. The virtual lapse rate of the atmosphere is 5 K gpkm^{-1}. Determine the stability of the atmosphere for this situation, when
    (a) $\delta q/\mathrm{d}z = 2$ cal kg^{-1} m^{-1}
    (b) $\delta q/\mathrm{d}z = 1$ cal kg^{-1} m^{-1}.
11. An air particle, warmer than the surroundings by 10 K, is moving upwards. Doing so it receives some heat due to absorption of long wave radiation. How far will the particle move, if the temperature lapse rate in the atmosphere is 6 K gpkm^{-1} and the particle receives heat according to the polytropic law $pv^n = $ const, with $n = 1.53$?
12. An air mass is ascending. The initial pressure is 1000 mb and the initial temperature is 290 K. Due to radiant heat exchange its potential temperature is increasing at the rate of 3.45% per kilometer. How much heat per unit mass does the air receive in the first 100 m of ascent?
13. A saturated layer 300 m thick is ascending at 2 m s^{-1} at the level of 850 mb. Its mean temperature is 20 °C. What is the maximum rate of precipitation that can be expected from it?
14. Consider a column of atmospheric air extending upwards to negligible pressures with a constant lapse rate $\gamma = 5$ K gpkm^{-1}. The pressure and temperature at the ground are: $p_0 = 1000$ mb, $T_0 = 300$ K. Compute its total internal energy, potential energy and enthalpy, per unit cross section. Neglect variation in the acceleration of gravity ($g = 9.8$ m s^{-2}); the air is unsaturated everywhere and can be considered as dry air.
15. Consider an adiabatic layer of dry air, extending from 500 to 400 mb. The temperature at the base is 0 °C.
    (a) What is the thickness of that layer, in gpm?
    (b) Compute the differences in specific internal energy u, enthalpy h, and entropy s, between parcels at the base and at the top.
    (c) What is the total internal energy of a column of that layer with unit cross section?