

SCA 5622
Météorologie synoptique et laboratoire de météo

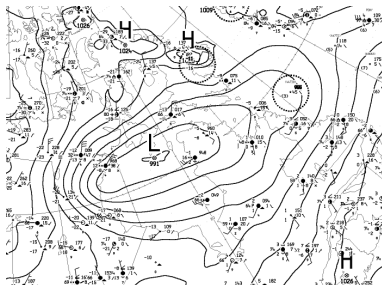
Le tourbillon

Le lundi 3 mars 2014
The logo of the Université du Québec à Montréal (UQAM) is displayed in blue.

Pourquoi étudier le tourbillon ?

Théorème de la circulation et
théorème de Stokes

$$C = \oint_C \vec{v} \cdot d\vec{l} = \iint_A (\nabla \times \vec{V}) \cdot \hat{n} dA$$



Quels sont les tourbillons que l'on doit considérer ?

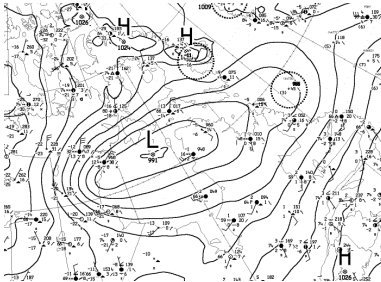
Tourbillon relatif

$$\zeta = \hat{k} \cdot (\nabla \times \vec{V}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

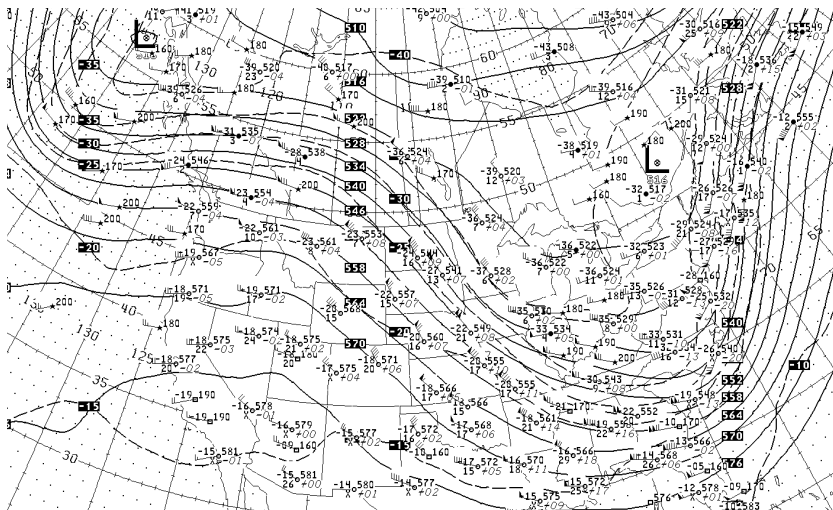
Tourbillon planétaire

$$\frac{C_p}{A} = \frac{2\Omega \cdot A \cdot \sin \phi}{A} = 2\Omega \sin \phi$$

$$\Rightarrow f = 2\Omega \sin \phi$$



$$\text{Tourbillon absolu : } \eta = \zeta + f$$



0000 UTC 5 March 2012 500 hPa Height/Temperature

$$\frac{d\zeta}{dt} = ?$$

1) Équations du mouvement en coordonnées cartésiennes :

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + F_{f_x} \quad (1a)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} + F_{f_y} \quad (1b)$$

2) On cherche $\left(\frac{\partial \zeta}{\partial t}\right)$, alors on applique $\frac{\partial(1b)}{\partial x} - \frac{\partial(1a)}{\partial y}$.

3) On obtient :

$$\begin{aligned} \frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} + (\zeta + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} \\ = \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \left(\frac{\partial F_{fy}}{\partial x} - \frac{\partial F_{fx}}{\partial y} \right) \end{aligned} \quad (2)$$

4) f varie seulement en y , on peut écrire que :

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z} = v \frac{\partial f}{\partial y} \quad (3)$$

$$\begin{aligned} \Rightarrow \frac{\partial \zeta}{\partial t} = & \underbrace{-\vec{v} \cdot \nabla_h \zeta - w \frac{\partial \zeta}{\partial z}}_1 - \underbrace{v \frac{\partial f}{\partial y}}_2 - \underbrace{(\zeta + f)\delta}_3 + \underbrace{\hat{k} \cdot \left(\frac{\partial \vec{v}}{\partial z} \times \nabla w \right)}_4 \\ & + \underbrace{\hat{k} \cdot \frac{(\nabla \rho \times \nabla p)}{\rho^2}}_5 + \underbrace{\hat{k} \cdot (\nabla \times \vec{F}_f)}_6 \end{aligned}$$