SCA 5622

Météorologie synoptique et laboratoire de météo

Le tourbillon

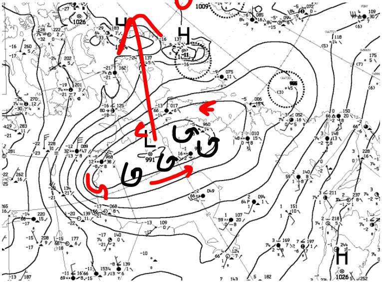
Le lundi 3 mars 2014

UQÀM

Pourquoi étudier le tourbillon ?

Théorème de la circulation et théorème de Stokes

$$C = \oint_C \vec{v} \cdot \vec{dl} = \iint_A (\nabla \times \vec{V}) \cdot \hat{n} dA$$



$\vec{v}$

$d\hat{l}$

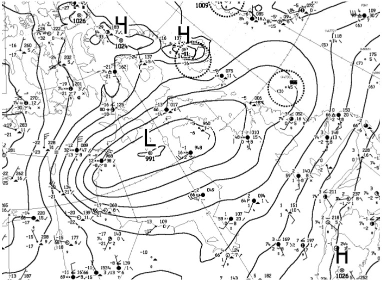
$c > 0$

$\sum \zeta =$ circulation

Quels sont les tourbillons que l'on doit considérer ?

Tourbillon relatif

$$\zeta = \hat{k} \cdot \left(\nabla \times \vec{V} \right) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

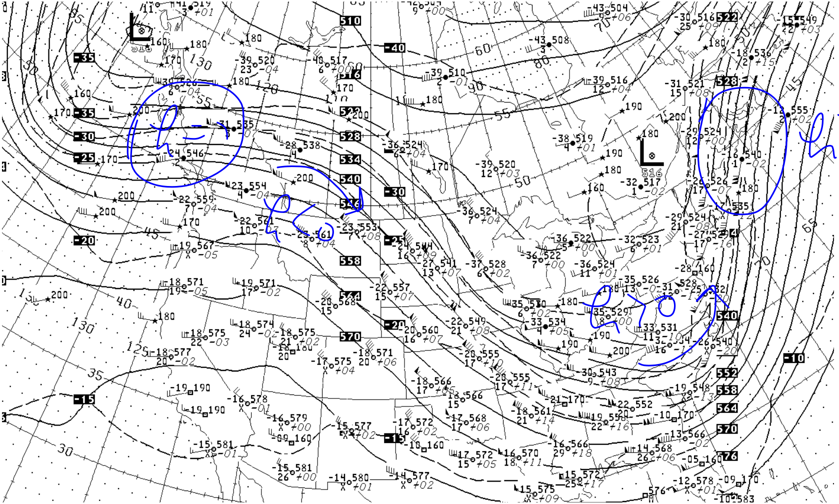


Tourbillon planétaire

$$\frac{C_p}{A} = \frac{2\Omega \cdot A \cdot \sin\phi}{A} = 2\Omega \sin\phi$$

$$\Longrightarrow f = 2\Omega \sin\phi$$

Tourbillon absolu : $\eta = \zeta + f$



0000 UTC 5 March 2012 500 hPa Height/Temperature

$$\zeta = V\,\partial\beta/\partial s - \partial V/\partial n$$

$$\frac{d\zeta}{dt} = ?$$

1) Équations du mouvement en coordonnées cartésiennes :

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_{f_x} \tag{1a}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_{f_y} \tag{1b}$$

2) On cherche $\left(\frac{\partial \zeta}{\partial t}\right)$, alors on applique $\frac{\partial (1b)}{\partial x} - \frac{\partial (1a)}{\partial y}$.

À faire en exercice

3) On obtient :

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} + (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y}$$
$$= \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right) + \left(\frac{\partial F_{f_y}}{\partial x} - \frac{\partial F_{f_x}}{\partial y}\right) \tag{2}$$

4) f varie seulement en y, on peut écrire que :

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} = v\frac{\partial f}{\partial y} \tag{3}$$

$$\Longrightarrow \frac{\partial \zeta}{\partial t} = \underbrace{-\vec{v}\cdot\nabla_h\zeta - w\frac{\partial \zeta}{\partial z}}_{1} \underbrace{- v\frac{\partial f}{\partial y}}_{2} \underbrace{-(\zeta + f)\delta}_{3} + \underbrace{\hat{k}\cdot\left(\frac{\partial \vec{v}}{\partial z}\times\nabla w\right)}_{4}$$
$$+ \underbrace{\hat{k}\cdot\frac{(\nabla\rho\times\nabla p)}{\rho^2}}_{5} + \underbrace{\hat{k}\cdot\left(\nabla\times\vec{F_f}\right)}_{6}$$

Voir notes pour la définition des termes.